h. Light travels at 2×10^8 m/s in glass and 2.25×10^8 m/s in water. Find the refractive index of water with respect to glass. Can total internal reflection happen for light going from glass into water? If so, find the critical angle.

i. **Think:** A small lamp lies on the bottom of a swimming pool 2 m deep (μ of water = 1.33). Light from it gets out of the water only through a circle on the surface. Explain why, and find the circle's radius.

Found a mistake? Please tell us.

Unit 13

Light

This unit online, with its interactives:
science-playground.com/s/9a13

Stand on the shore of Rara Lake on a still morning and look down. The water is so clear that you can see fish gliding over the stones, and the stones look close enough to touch. Wade in, though, and the water comes up much higher on your legs than you expected. The bottom was deeper than it looked. A fisherman with a spear learns the same lesson: aim straight at the fish you see and you miss, because the fish is not quite where it seems to be.

Light is playing a trick here. It travels in straight lines, but where it passes from water into air it bends, and your eyes and brain don't know that it bent. The same bending lets a doctor look inside your stomach through a thin cable, carries the internet into Nepal through glass threads thinner than a hair, and makes a diamond flash with colored fire. This unit is about how much light bends, a rule that lets you work it out with a calculator, and what happens when light can't get out at all.

You already know from Grade 7 that light travels in straight lines and that a mirror reflects it by the law of reflection: the angle of reflection equals the angle of incidence Grade 7. When light meets the surface of a transparent material such as water or glass, most of it doesn't bounce back. It goes on into the new material, and something changes: its **speed**.

In empty space, and almost exactly in air, light travels at about 3×10^8 m/s, three hundred thousand kilometers every second. In water it travels at only about three-quarters of that speed, in glass about two-thirds, and in diamond well under half. A material in which light travels more slowly is called **optically denser**; one in which it travels faster is **optically rarer**. Glass is optically denser than water, and water is optically denser than air.



Figure 13.1 *Fish in the clear shallows of Rara Lake, Mugu. The fish and the bottom look nearer the surface than they really are.*
Photo: Redpandamoon, CC BY-SA 4.0

CAREFUL

Optically denser is not the same as heavier

"Denser" here is about the speed of light, not about mass. Kerosene floats on water, so it is less dense than water in mass per liter (about 0.8 g/cm³ against 1 g/cm³). Yet light travels more slowly in kerosene than in water: kerosene is optically **denser**. When a question says "denser medium" about light, it always means optically denser.

13.1 Refraction

What refraction is

Put a pencil in a glass of water at a slant: it looks broken at the surface. Shine a laser pointer into a tank of water from above at a slant and look from the side: the beam changes direction as it enters the water. This change of direction is refraction.

Refraction of light is the change in the direction of light when it passes at a slant from one transparent medium into another, because its speed changes.

ii. Ordinary reflection (from a mirror) and total internal reflection

iii. Real depth and apparent depth

4. Answer the following questions

a. Define refraction of light, and state the two laws of refraction.

b. What is the absolute refractive index of a medium? How is it related to the speed of light in the medium?

c. What is total internal reflection? Write its two conditions. Describe a simple experiment to demonstrate it.

d. Derive the relation between the critical angle and the refractive index of a medium.

e. Describe how you would find the refractive index of a glass slab in the laboratory. Draw a labeled diagram.

f. Show that real depth ÷ apparent depth = μ for an object under water seen from straight above.

g. Write two uses of total internal reflection, and explain how one of them works.

5. Numerical problems

a. A ray of light passes from air into a block of glass. The angle of incidence is 45° and the angle of refraction is 28°. Find the refractive index of the glass.

b. Light travels at 2.25×10^8 m/s in a liquid. Find the refractive index of the liquid and its critical angle. ($c = 3 \times 10^8$ m/s)

c. A ray in air strikes the surface of glycerine ($\mu = 1.47$) at an angle of incidence of 60°. Find the angle of refraction.

d. The critical angle of a transparent material is 35°. Find its refractive index and the speed of light in it.

e. Rizwan looks straight down into a water tank 1.2 m deep. How deep does the tank look to him? ($\mu = 4/3$)

f. A mark on paper, seen through a glass slab 3.6 cm thick laid on it, seems to be raised by 1.2 cm. Find the refractive index of the glass.

g. Anisha uses a traveling microscope to find the refractive index of a liquid. Focused on a scratch on the bottom of an empty beaker, it reads 1.12 cm. With liquid poured in, it must be raised to 2.08 cm to see the scratch clearly. Focused on a little powder floating on the liquid's surface, it reads 4.90 cm. Find the refractive index of the liquid.

c. The angle of incidence in a denser medium for which the angle of refraction in a rarer medium is 90° is called:

a the angle of deviation
b the angle of emergence
c the critical angle
d the angle of reflection

d. The critical angle of glass is 42°. Light inside glass meets the glass–air surface at 50°. It will be:

a refracted into the air, bent toward the normal
b refracted into the air, bent away from the normal
c totally reflected back into the glass
d absorbed at the surface

e. Which relation is correct for a medium of refractive index μ in air, with critical angle c?

a $\mu = \sin c$
b $\mu = \frac{1}{\sin c}$
c $\mu = \frac{1}{\cos c}$
d $\mu = \tan c$

f. The refractive index of water is 4/3. A coin at the bottom of a tank of water 2 m deep appears, from straight above, to be at a depth of:

a 2.67 m
b 2 m
c 1.5 m
d 0.67 m

g. Optical fibers carry light by:

a refraction at their ends only
b repeated total internal reflection at the edge of their core
c reflection from a silver coating
d diffraction round the bends

h. In the glass slab experiment, the angle of emergence is:

a equal to the angle of refraction
b equal to the angle of incidence
c always 90°
d greater than the angle of incidence

2. Give reasons

a. A pond looks shallower than it really is.

b. A diamond sparkles much more than a glass copy of the same shape.

c. Light passing along the normal from air into glass does not bend.

d. Total internal reflection cannot happen when light goes from air into water.

e. An empty test tube held at a slant in a beaker of water looks silvery from above, but not once it is filled with water.

3. Differentiate between

i. Angle of incidence and critical angle

To describe refraction, we need a few words, shown in Figure 13.3.

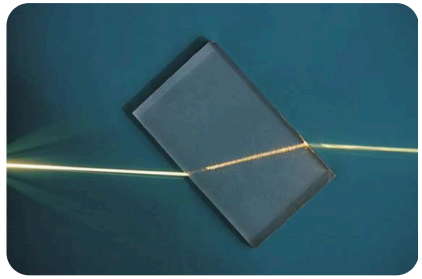


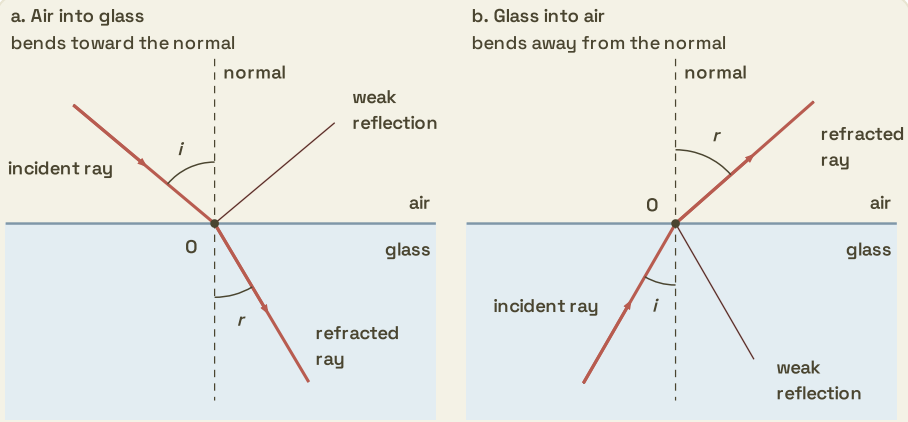
Figure 13.2 *A narrow beam of light passing through a clear rectangular block. It bends where it enters the block and bends back where it leaves.*
Photo: ajizai, Public domain

- The **incident ray** is the light coming toward the surface.
- The **point of incidence** (O) is where it meets the surface.
- The **normal** is a line drawn at right angles to the surface at the point of incidence. It is an imaginary line, drawn dashed.
- The **refracted ray** is the light that goes on into the second medium.
- The **angle of incidence** (i) is the angle between the incident ray and the normal.
- The **angle of refraction** (r) is the angle between the refracted ray and the normal.

Both angles are always measured from the **normal**, never from the surface. A ray that strikes the surface at 30° to the surface makes an angle of incidence of 60°.

The rule for the direction is simple:

- Light going from a **rarer** into a **denser** medium (air into water, water into glass) bends **toward the normal**: $r < i$.
- Light going from a **denser** into a **rarer** medium (glass into air, water into air) bends **away from the normal**: $r > i$.
- Light that meets the surface **along the normal** ($i = 0^\circ$) goes straight on without bending, although it still slows down or speeds up.



IN SHORT

- **Refraction** is the change in direction of light passing at a slant from one medium into another, because its speed changes. Light going into an optically denser medium bends toward the normal; into a rarer one, away from it; along the normal, not at all.
- **Laws of refraction:** the incident ray, refracted ray and normal lie in one plane; and $\frac{\sin i}{\sin r}$ is constant for two given media (**Snell's law**).
- **Refractive index:** $\mu = \frac{\sin i}{\sin r} = \frac{c}{v}$, with no unit; relative refractive index ${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$; in general $\mu_1 \sin i = \mu_2 \sin r$; by reversibility ${}_2\mu_1 = \frac{1}{{}_1\mu_2}$.
- The **critical angle** is the angle of incidence in the denser medium for which the angle of refraction is 90°: $\sin c = \frac{1}{\mu}$ (in air), $\sin c = \frac{\mu_2}{\mu_1}$ between two media.
- **Total internal reflection** happens when light goes from a denser into a rarer medium at more than the critical angle: all of it is reflected. Uses: diamonds' sparkle, optical fibers, endoscopes, prisms in binoculars and periscopes.
- The refractive index of glass is found by tracing rays through a slab with pins: $\mu = \frac{\sin i}{\sin r}$. The emergent ray is parallel to the incident ray, shifted sideways.
- Looking straight down, $\mu = \frac{\text{real depth}}{\text{apparent depth}}$; apparent shift = real depth $\times$ $(1 - \frac{1}{\mu})$.

Exercise

1. Choose the correct option

a. When light passes from air into glass at a slant, it:

- **a** speeds up and bends away from the normal
- **b** slows down and bends toward the normal
- **c** slows down and goes on in a straight line
- **d** speeds up and bends toward the normal

b. The unit of refractive index is:

- **a** m/s
- **b** degree
- **c** s/m
- **d** it has no unit

WORKED EXAMPLE

Two liquids in one jar

A jar holds 20 cm of water ($\mu = 4/3$) with 12 cm of kerosene ($\mu = 1.44$) floating on top. A coin lies on the bottom. How deep does it look from straight above?

Answer

Each layer makes its own thickness look shorter by its own refractive index, and the apparent depths add up:

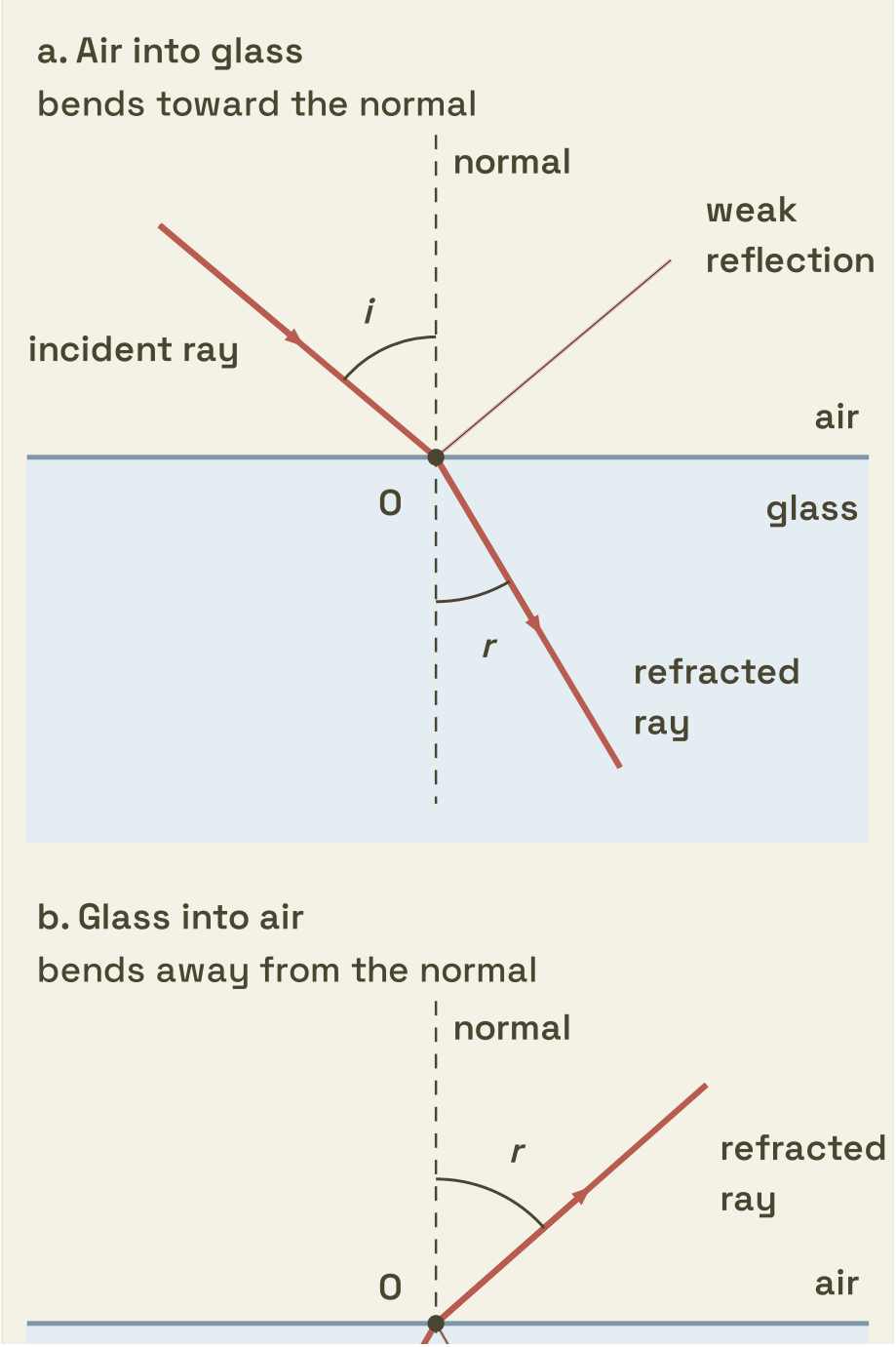
$$\text{apparent depth} = \frac{12}{1.44} + \frac{20}{4/3} = 8.33 + 15 = 23.3 \text{ cm}$$

About 23.3 cm, though the coin is really 32 cm down.

Check 6

A coin lies at the bottom of a beaker of water 16 cm deep ($\mu = 4/3$). Looking straight down, how far does it seem to be raised?

a 12 cm
b 4 cm
c 21.3 cm
d 5.3 cm



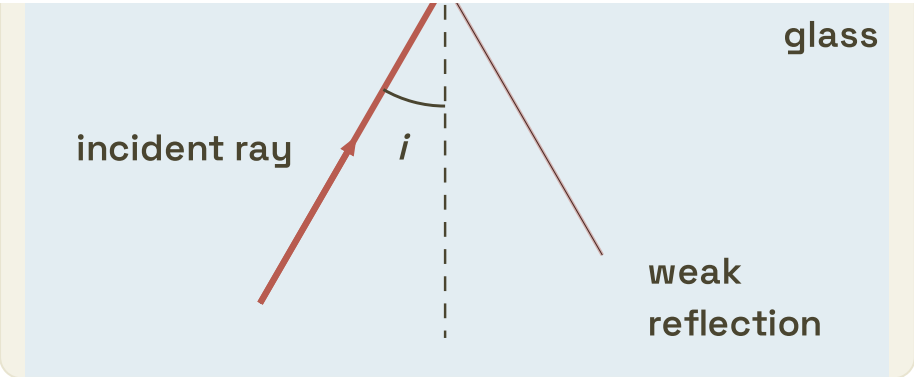


Figure 13.3 *Refraction, drawn with glass of refractive index 1.5. a. Going from air into glass, the light slows down and bends toward the normal: an angle of incidence of 50° gives an angle of refraction of 31°. b. Going from glass into air, it speeds up and bends away from the normal: 30° gives 49°. A small part of the light (here about 6%) is reflected at the surface as well.*

Why a change of speed bends light

Imagine a line of students marching side by side across the school ground, arms linked, and coming at a slant to a patch of soft sand where everyone walks more slowly. The student at one end reaches the sand first and slows down, while the others keep their speed for a moment longer. The line swings round, and by the time all of them are on the sand it is marching in a new direction, turned toward the line square to the edge of the sand. Coming out of the sand at a slant, the first to reach firm ground speeds up first, and the line swings back the other way.

A beam of light behaves in the same way. Its front edge is like the marching line: the side that enters the slower medium first is slowed first, so the whole front turns. If the line hits the edge of the sand square on, everyone slows at the same moment, and the line goes straight on. That is why light along the normal doesn't bend.

Check 1

A ray of light passes from water into air at an angle of incidence of 30°. Which is true of the angle of refraction?

- **a** it is less than 30°, because the light bends toward the normal
- **b** it is more than 30°, because the light bends away from the normal
- **c** it is exactly 30°, because the light goes straight on
- **d** it is 60°, because angles are measured from the surface

The laws of refraction

People had seen light bend in water since ancient times, but finding the exact rule took more than a thousand years. Measurements of i and r for many angles give two

WORKED EXAMPLE

How far does the mark rise?

A glass block 7.5 cm thick ($\mu = 1.5$) is placed on a printed page. By how much does the print seem to be raised, seen from above?

Answer

$$\text{apparent depth} = \frac{7.5}{1.5} = 5 \text{ cm}$$

$$\text{apparent shift} = 7.5 - 5 = 2.5 \text{ cm} \quad \text{or} \quad 7.5 \times \left(1 - \frac{1}{1.5}\right) = 2.5 \text{ cm}$$

The print seems raised by 2.5 cm, a third of the block's thickness.

WORKED EXAMPLE

The traveling microscope

Sujata finds the refractive index of a glass slab with a traveling microscope, a microscope that slides up and down a scale. She focuses on a mark on the table and reads 2.50 cm. She puts the slab over the mark and focuses on the mark through it: 3.80 cm. Then she sprinkles a little chalk powder on top of the slab and focuses on it: 6.40 cm. Find the refractive index.

Answer

Real depth of the mark below the top of the slab $= 6.40 - 2.50 = 3.90$ cm (the slab's thickness).

Apparent depth $= 6.40 - 3.80 = 2.60$ cm (how far below the top the mark seems to be).

$$\mu = \frac{\text{real depth}}{\text{apparent depth}} = \frac{3.90}{2.60} = 1.5$$

$\mu = 1.5$. The microscope had to be raised by 1.30 cm to focus on the mark through the glass: that is the apparent shift.

Worked examples

WORKED EXAMPLE

How deep is the pool?

A swimming pool is filled with water 2.4 m deep. How deep does it look to someone looking straight down? (μ of water = 4/3)

Answer

$$\text{apparent depth} = \frac{\text{real depth}}{\mu} = \frac{2.4}{4/3} = 2.4 \times \frac{3}{4} = 1.8 \text{ m}$$

It looks 1.8 m deep, 60 cm less than it is.

WORKED EXAMPLE

A mark under a glass slab

A mark on a sheet of paper is seen through a glass slab 6 cm thick laid on it, looking straight down. The mark appears 4 cm below the top of the slab. Find the refractive index of the glass.

Answer

The real depth of the mark below the top of the slab is the slab's thickness, 6 cm; its apparent depth is 4 cm.

$$\mu = \frac{\text{real depth}}{\text{apparent depth}} = \frac{6}{4} = 1.5$$

μ = 1.5.

WORKED EXAMPLE

How deep is the fish really?

A fish in a pond appears to be 60 cm below the surface, seen from straight above. How deep is it really? (μ of water = 4/3)

Answer

$$\text{real depth} = \mu \times \text{apparent depth} = \frac{4}{3} \times 60 = 80 \text{ cm}$$

80 cm. The fish is 20 cm deeper than it seems.

laws.

1. The incident ray, the refracted ray and the normal at the point of incidence all lie **in the same plane**.
2. For a given pair of media and a given color of light, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a **constant**:

$$\frac{\sin i}{\sin r} = \text{constant}$$

The second law is **Snell's law**. The constant is called the **refractive index** of the second medium with respect to the first. In this book it is written μ (the Greek letter mu); many books write *n*. Both mean the same thing.

The first law says that refraction never bends light up out of the page or down into it: if the incident ray and the normal are drawn on paper, the refracted ray is on the same paper. The second law is the one you calculate with.

Notice that it is the **sines** of the angles that keep a fixed ratio, not the angles themselves. For light going from air into glass, $i = 30°$ gives $r = 19.5°$, but $i = 60°$ gives $r = 35.3°$, not 39°. The angles are not in a fixed ratio; their sines are: sin 30° ÷ sin 19.5° and sin 60° ÷ sin 35.3° are both 1.5.

A STORY FROM SCIENCE

Is light faster or slower in water?

Snell's law was found more than once. The mathematician Ibn Sahl drew it correctly at the court in Baghdad around 984; the English astronomer Thomas Harriot found it again in 1602 and the Dutch mathematician Willebrord Snellius in 1621, neither of them publishing it; and the French philosopher René Descartes published it in 1637. But **why** does light bend? Isaac Newton thought light was a stream of tiny particles, pulled toward water as they entered it, so they should go **faster** in water. The Dutch scientist Christiaan Huygens thought light was a wave, which bends because it goes **slower** in water. Both theories gave the same law of sines, and for 150 years nobody could measure the speed of light in water to settle it. On 27 April 1850, the French physicist Léon Foucault sent light through a tube of water and back to a mirror spinning hundreds of times a second, and measured that light was slower in water than in air. Hippolyte Fizeau and Louis Breguet found the same seven weeks later. The wave theory had won.

Refractive index

The refractive index of a medium tells how much it slows light down, and so how strongly it bends light. When light passes from a vacuum (or air) into a medium, the refractive index of the medium is the ratio of the speed of light in a vacuum, c, to its speed in the medium, v:

$$\mu = \frac{\sin i}{\sin r} = \frac{\text{speed of light in vacuum}}{\text{speed of light in the medium}} = \frac{c}{v}$$

This is the **absolute refractive index** of the medium. Since it is one speed divided by another, it has **no unit**. Air's refractive index is 1.0003, so close to 1 that we take it as 1, and the refractive index "with respect to air" is the same as the absolute one.

Medium	Refractive index (μ)	Speed of light (m/s)
air	1.0003 (take 1)	3.0×10^8
ice	1.31	2.29×10^8
water	1.33	2.26×10^8
kerosene	about 1.44	2.08×10^8
glycerine	1.47	2.04×10^8
ordinary glass	1.5	2.0×10^8
diamond	2.42	1.24×10^8

A higher refractive index means slower light and more bending. The refractive index of a material also changes slightly with the color of the light: violet light is slowed a little more than red, which is why a prism splits white light into colors. The values in tables like this one are for yellow light. Kerosene is a mixture whose make-up varies, so its refractive index varies a little too, from about 1.43 to 1.45; problems use 1.44.

DID YOU KNOW?

Water's 4/3

The refractive index of water is 1.333, so close to $\frac{4}{3}$ that many problems use 4/3. Then the speed of light in water is exactly three-quarters of its speed in air, and the numbers often come out neat.

Relative refractive index

Light does not always start in air. When it goes from medium 1 (water, say) into medium 2 (glass), Snell's law still holds, and the constant is the **relative refractive**

CAREFUL

Only when looking straight down

The formula $\mu = \frac{\text{real depth}}{\text{apparent depth}}$ holds only for light very close to the normal, that is, looking straight down. Seen at a slant, a pool looks shallower still, and the bottom seems to tilt up toward the far side. That is why a pool or a river can look knee-deep from the bank when it is well over your head: the farther away you look, the shallower it seems.

Real and apparent depth around us

Pools, ponds and rivers look shallower than they are. A pool 2 m deep looks only 1.5 m deep from straight above, and less from the side. Every monsoon, people drown in Nepal's rivers and ponds, and water that looks shallower than it is adds to the danger. Never jump or wade into water whose depth you haven't checked with a stick.

The fisherman's aim. A fish seen from the bank appears higher, and nearer, than it is. A spear aimed straight at the fish's image passes above it; fishermen learn to aim below where the fish appears.

Seen from under water. The light works the other way too. To a swimmer under water looking up, a person standing at the pool's edge looks taller and farther up than they are, because light from the air bends toward the normal as it enters the water.

A thick glass slab. Print seen through a thick glass block laid on a page looks raised, closer to the top of the block: for a 6 cm glass block ($\mu = 1.5$) the print appears 4 cm below the top, raised by 2 cm.

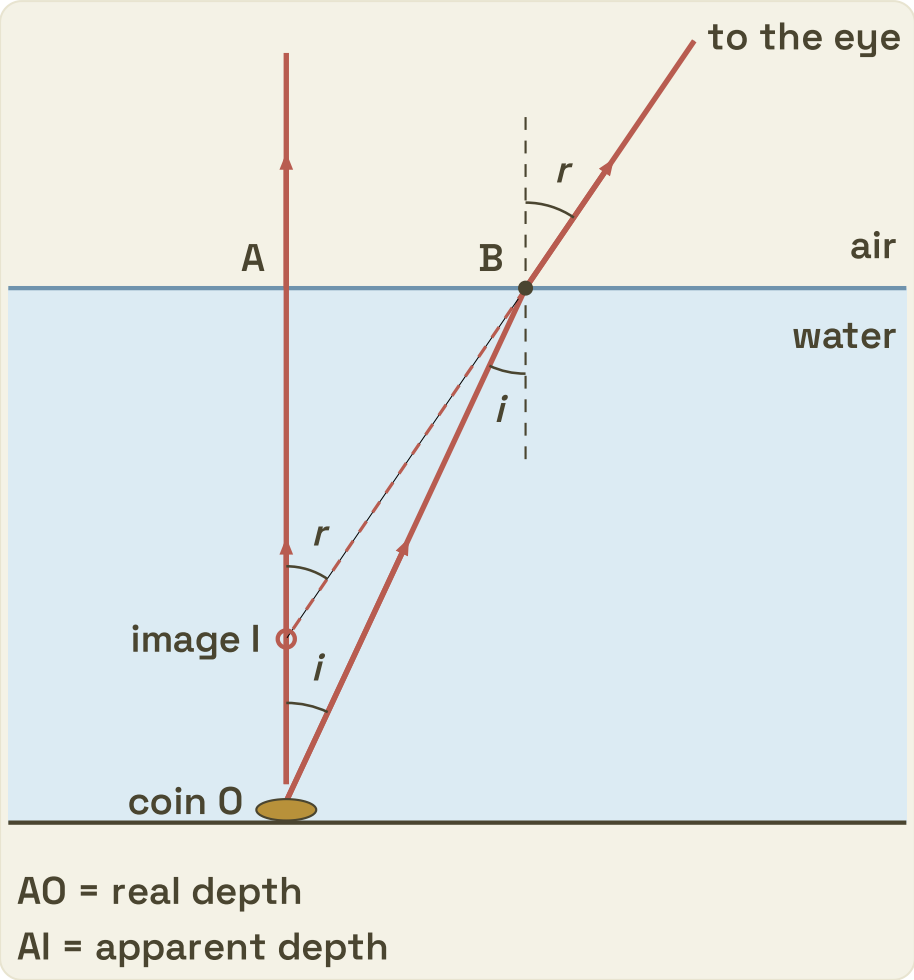


Figure 13.10 *Real and apparent depth. Light from the coin O that meets the surface at B bends away from the normal; traced back, it seems to come from I, the image, higher up. The angles are drawn large so they can be seen: looking straight down, B is very close to A, and AO ÷ AI is the refractive index, 1.33 for water.*

index of medium 2 with respect to medium 1, written ${}_1\mu_2$:

$$ {}_1\mu_2 = \frac{\sin i}{\sin r} = \frac{\text{speed of light in medium 1}}{\text{speed of light in medium 2}} = \frac{v_1}{v_2} $$

Since $v_1 = \frac{c}{\mu_1}$ and $v_2 = \frac{c}{\mu_2}$, the c's cancel:

$$ {}_1\mu_2 = \frac{\mu_2}{\mu_1} $$

So the refractive index of glass with respect to water is ${}_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{1.5}{1.33} = 1.13$. A relative refractive index can be **less than 1**: that just means the light is going into a faster medium, as from glass into water (${}_g\mu_w = \frac{1.33}{1.5} = 0.887$).

Multiplying both sides of $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$ out gives Snell's law in a form that works for any two media at once:

$$ \mu_1 \sin i = \mu_2 \sin r $$

This is often the quickest way to solve a problem: put the refractive index of each medium beside the sine of the angle in it.

The principle of reversibility

If a ray of light is sent back along the path it came, it retraces that path exactly. A ray going from air into glass at 50° is refracted at 30.7°; send light back along that refracted ray from inside the glass at 30.7°, and it comes out into the air at exactly 50°. This is the **principle of reversibility** of light.

Applied to Snell's law, it says that the refractive index from medium 2 to medium 1 is the reciprocal of the one from 1 to 2:

$$ {}_2\mu_1 = \frac{1}{{}_1\mu_2} \qquad \text{for example} \qquad {}_g\mu_a = \frac{1}{{}_a\mu_g} = \frac{1}{1.5} = 0.667 $$

Worked examples

WORKED EXAMPLE

The refractive index from two angles

A ray of light passes from air into a glass block. The angle of incidence is 60° and the angle of refraction is 35.3°. Find the refractive index of the glass.

Answer

$i = 60°$, $r = 35.3°$

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 35.3°} = \frac{0.866}{0.578} = 1.50$$

The refractive index of the glass is 1.5 (no unit).

WORKED EXAMPLE

Finding the angle of refraction

Light in air meets the surface of water ($\mu = 1.33$) at an angle of incidence of 45°. Find the angle of refraction.

Answer

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 45°}{1.33} = \frac{0.707}{1.33} = 0.532$$

$$r = \sin^{-1}(0.532) = 32.1°$$

The angle of refraction is about 32°. It is smaller than 45°: going into a denser medium, the light bends toward the normal, as it should.

WORKED EXAMPLE

The speed of light in glass and in diamond

The speed of light in air is 3×10^8 m/s. Find its speed in glass ($\mu = 1.5$) and in diamond ($\mu = 2.42$).

Answer

From $\mu = \frac{c}{v}$, $v = \frac{c}{\mu}$.

$$v_{\text{glass}} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ m/s} \qquad v_{\text{diamond}} = \frac{3 \times 10^8}{2.42} = 1.24 \times 10^8 \text{ m/s}$$

Light travels at 2×10^8 m/s in glass and 1.24×10^8 m/s in diamond.

be, its image, is its **apparent depth**. For anything under water seen from above, the apparent depth is less than the real depth.

The reason is refraction at the surface. Light from the coin travels up through the water and bends away from the normal as it passes into the air. Your eye can't tell that the light bent. It traces the light straight back, and sees the coin where the bent light seems to come from, higher up.

Real depth ÷ apparent depth = μ

In Figure 13.10, a coin O lies on the bottom of the water, and A is the point on the surface straight above it. Light from O that goes straight up, along OA, meets the surface along the normal and isn't bent. Light from O that goes along OB meets the surface at B, at an angle of incidence i, and leaves bent away from the normal at angle r. Traced back, the ray from B meets the line OA at I. That is where the eye sees the coin: AO is the real depth and AI the apparent depth.

The angle at O between OA and OB equals i, and the angle at I between IA and IB equals r (each is the same as the angle at B, since OA and the normal at B are parallel). In the right-angled triangles OAB and IAB,

$$\tan i = \frac{AB}{AO} \qquad \tan r = \frac{AB}{AI}$$

For light going from water into air, Snell's law gives $\frac{\sin r}{\sin i} = \mu$, the refractive index of water. When we look **straight down**, only rays very close to the normal reach the eye, so i and r are very small, and for small angles the sine and tangent of an angle are almost equal. Then

$$\mu = \frac{\sin r}{\sin i} \approx \frac{\tan r}{\tan i} = \frac{AB/AI}{AB/AO} = \frac{AO}{AI}$$

$$\mu = \frac{\text{real depth}}{\text{apparent depth}}$$

Looking straight down into water, things appear at $\frac{1}{1.33}$, about three-quarters, of their real depth. The amount by which the object seems to rise is the **apparent shift** (or normal shift):

$$\text{apparent shift} = \text{real depth} - \text{apparent depth} = \text{real depth} \times \left(1 - \frac{1}{\mu}\right)$$

$$\frac{\sin i}{\sin r} \times \frac{\sin r}{\sin e} = \mu \times \frac{1}{\mu} = 1 \qquad \text{so} \qquad \sin e = \sin i, \quad e = i$$

The light leaves going in the same direction it came in. It is only shifted sideways, by the **lateral shift** (or lateral displacement), the perpendicular distance between the emergent ray and the incident ray produced. A thicker slab, a bigger angle of incidence or a higher refractive index gives a bigger shift. This is why a line seen through a thick glass slab at a slant looks broken where the glass begins, but still points the same way.

DID YOU KNOW?

Working out the shift

For a slab of thickness t, the lateral shift is $t \sin(i - r) \div \cos r$. For the 6 cm slab in Figure 13.9, at $i = 40°$ and $r = 25.4°$, it is $6 \times \sin 14.6° \div \cos 25.4° = 1.68$ cm. You don't need this formula for the exam, but you can check it on your own drawing.

Other ways to find a refractive index

The pin method is not the only one. With a **semicircular block**, as in the widget in 13.1, the light leaving through the curved side meets it along a radius and isn't bent there, so both angles can be read straight off a protractor; and sending the beam in through the curved side gives the critical angle, and so $\mu = \frac{1}{\sin c}$. A third way uses real and apparent depth, the subject of the next section: $\mu = \frac{\text{real depth}}{\text{apparent depth}}$, measured with a traveling microscope.

Check 5

In the glass slab experiment, the emergent ray is:

a along the incident ray produced

b parallel to the incident ray, but shifted sideways

c parallel to the refracted ray inside the slab

d at right angles to the incident ray

13.4 Real and apparent depth

Meaning

Drop a coin into a bucket of water and look down at it. It seems to lie higher than it does, as if the bucket had a shallower bottom. The distance from the surface down to where the object really is, is its **real depth**. The distance down to where it seems to

WORKED EXAMPLE

From water into glass

Light in water ($\mu = 1.33$) meets a glass plate ($\mu = 1.5$) at an angle of incidence of 40°. Find the refractive index of glass with respect to water, and the angle of refraction in the glass.

Answer

$$_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{1.5}{1.33} = 1.13$$

Using $\mu_1 \sin i = \mu_2 \sin r$:

$$\sin r = \frac{\mu_w \sin i}{\mu_g} = \frac{1.33 \times \sin 40°}{1.5} = \frac{1.33 \times 0.643}{1.5} = 0.570 \qquad r = 34.7°$$

The relative refractive index is 1.13, and the light is refracted at about 35°. The bend is small, because water and glass slow light down by similar amounts.

Check 2

The refractive index of a liquid is 1.25. What is the speed of light in it? ($c = 3 \times 10^8$ m/s)

a 3.75×10^8 m/s

b 2.4×10^8 m/s

c 1.25×10^8 m/s

d 3×10^8 m/s

13.2 Total internal reflection of light

From a denser into a rarer medium

When light goes from glass into air, it bends away from the normal, so the angle of refraction is always **bigger** than the angle of incidence. Make i bigger, and r grows faster still. But r can't be more than 90°: at 90°, the refracted ray skims along the surface. What happens then?

Go back to the widget above and choose **Block into air**. Move the angle of incidence up slowly from 0° and watch three things: the refracted ray swinging toward the surface, the reflected ray getting stronger, and the moment the refracted ray disappears.

The critical angle

The **critical angle** (c) is the angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90°.

For glass in air, it is 41.8°. Below it, light gets out, bent away from the normal. Exactly at it, the refracted ray skims along the surface. Beyond it, no light gets out at all (Figure 13.4).

The critical angle and the refractive index

Snell's law gives the critical angle at once. For light going from a medium of refractive index μ into air, $\mu_1 \sin i = \mu_2 \sin r$ becomes, at the critical angle ($i = c, r = 90^\circ$):

$$\mu \sin c = 1 \times \sin 90^\circ = 1 \qquad \text{so} \qquad \sin c = \frac{1}{\mu}$$

The bigger the refractive index, the smaller the critical angle.

Medium (in air)	μ	Critical angle
water	1.33	48.8°
kerosene	about 1.44	about 44°
glycerine	1.47	42.9°
glass	1.5	41.8°
diamond	2.42	24.4°

Between two media that aren't air, the same reasoning gives $\sin c = \dfrac{\mu_2}{\mu_1}$, where μ_1 is the denser medium's index and μ_2 the rarer one's. Light in glass meeting water has a critical angle of $\sin^{-1} \frac{1.33}{1.5} = 62.5°$.

Total internal reflection

When light in a denser medium meets the surface of a rarer medium at an angle of incidence **greater than the critical angle**, none of it is refracted: all of it is reflected back into the denser medium, obeying the laws of reflection. This is **total internal reflection**. It is "internal" because the light stays inside the denser medium, and "total" because all of it is reflected.

Two conditions are needed for total internal reflection:

1. The light must be going from an optically **denser** medium into an optically **rarer** one.
2. The angle of incidence must be **greater than the critical angle** for the pair of media.

Ordinary reflection from a mirror is never total: even a good silvered mirror absorbs a few percent of the light and slowly tarnishes. Total internal reflection reflects all

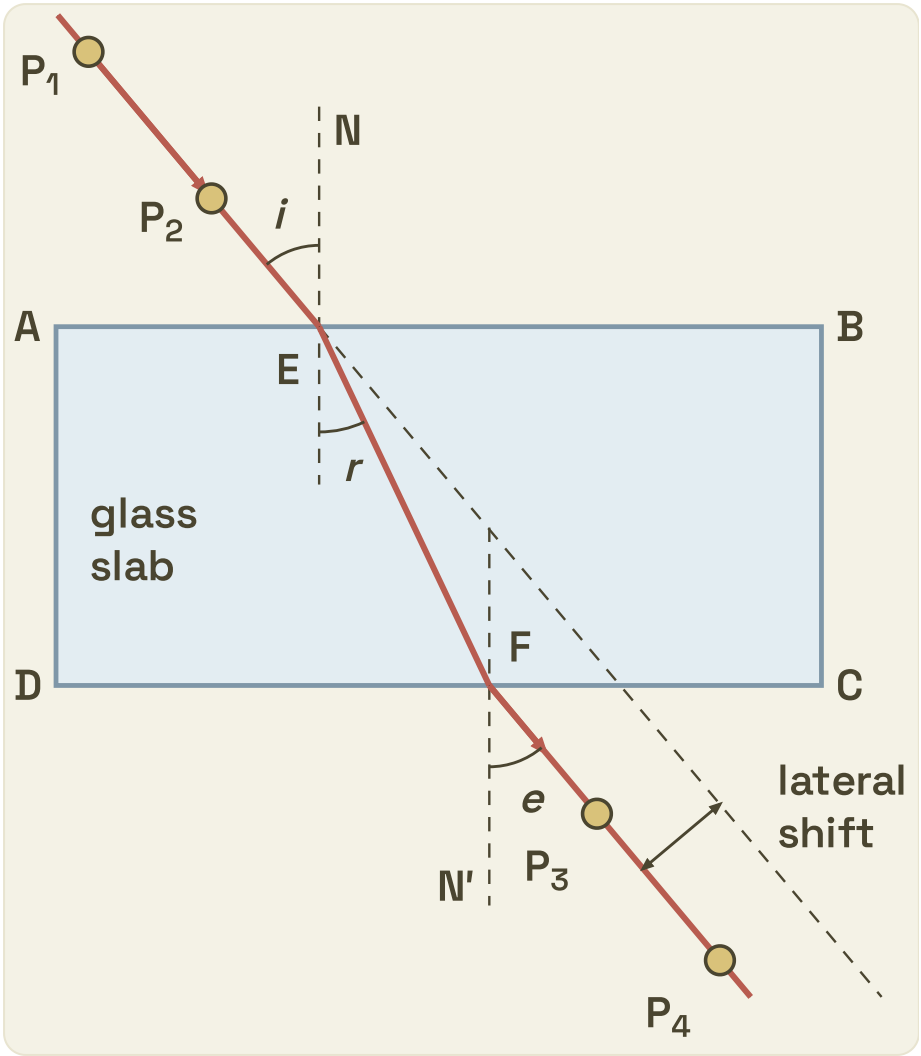


Figure 13.9 *The pin method, seen from above. The incident ray P_1P_2 meets face AB at E at an angle of incidence of 40°. Inside the glass ($\mu = 1.5$) the ray EF is refracted at 25.4°. It leaves face DC at F, where pins P_3 and P_4 mark the emergent ray, parallel to the incident ray but shifted sideways (the lateral shift, 1.68 cm for this 6 cm slab).*

The emergent ray is parallel to the incident ray

At face AB, light goes from air into glass: $\frac{\sin i}{\sin r} = \mu$. At face DC it goes from glass into air, with angle of incidence r (the two normals are parallel, so the angle inside is the same at both faces) and angle of emergence e: $\frac{\sin r}{\sin e} = \frac{1}{\mu}$. Multiplying the two,

DID YOU KNOW?

The whole sky in a circle

Lie at the bottom of a clear swimming pool and look up. Light from the whole sky, from the horizon on every side to straight overhead, comes into the water bent toward the normal, at no more than the critical angle, 48.8°. So you see the entire sky squeezed into a bright circle directly above you, about 97° across. Outside that circle the surface acts as a mirror and shows the bottom of the pool. Divers call it Snell's window.

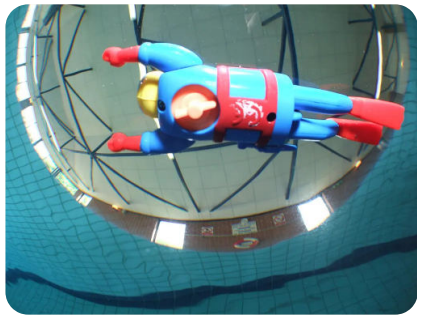


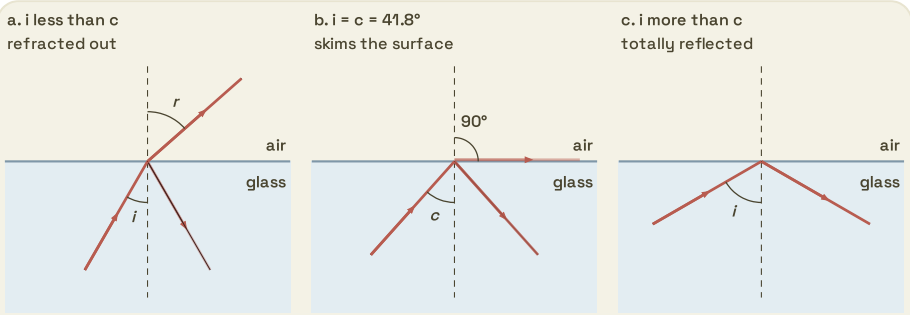
Figure 13.8 *Snell's window, from the bottom of an indoor pool, with a toy diver. The whole ceiling of the hall appears inside the bright circle; outside it, the surface acts as a mirror and reflects the pool's tiles.*

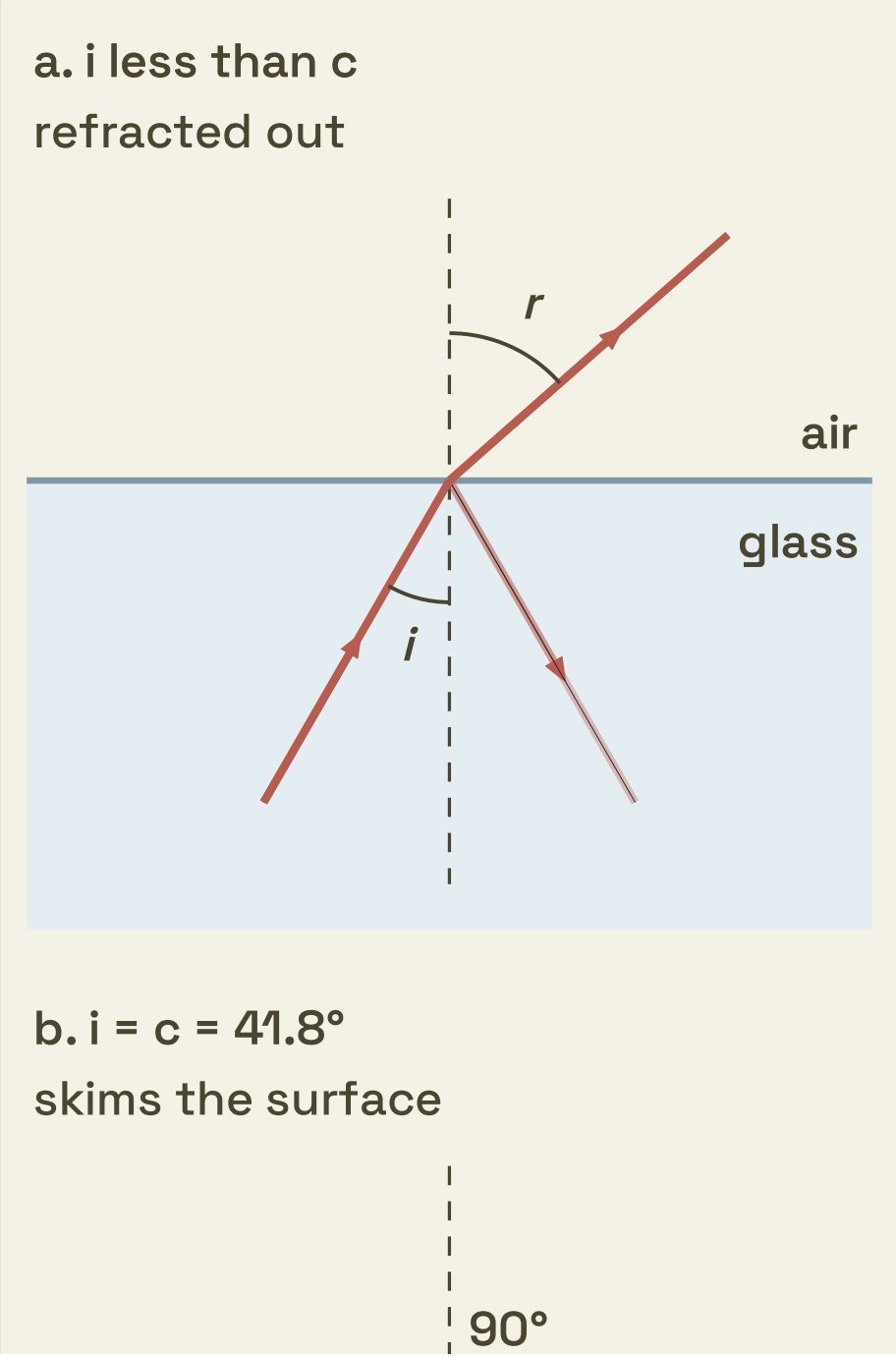
Photo: petebw, CC BY-SA 2.0

13.3 Determination of refractive index of glass

The refractive index of a glass slab can be found in the laboratory by tracing a ray through it with pins. The pins mark the ray's path on paper: when you look through the glass and see two pins, one behind the other, in a straight line, the light from them reaches your eye along the line you are looking.

Getting a good result. Stand the pins upright and well apart (pins close together give a line whose direction is uncertain). Keep the angle of incidence between about 30° and 60°: at very small angles the angles are tiny and hard to read to the same accuracy, and at very large ones the light reflected from AB makes the pins hard to see. Use a sharp pencil and a fine line, and look at the feet of the pins.





flected, again and again, thousands of times a meter, and follows the fiber round bends for many kilometers with very little loss. Fibers carry telephone calls and the internet as flashes of light. Most of Nepal's internet reaches the world through fiber cables across the border with India, at Birgunj, Bhairahawa and Biratnagar among other places, and since 12 January 2018 also through a cable from China at Rasuwagadhi.

Endoscopes. A doctor's endoscope is a bundle of thousands of optical fibers in a flexible tube. Some carry light down into the stomach or the knee; others, laid in exactly the same order at both ends, carry the picture back, each fiber one dot of it. It lets doctors see inside the body without large cuts.

Prisms that turn light. A glass prism with angles of 90°, 45° and 45° turns light through 90° or 180°: the light meets the long face at 45°, more than glass's 41.8°, and is totally reflected. Periscopes and binoculars use such prisms instead of mirrors, since they reflect all the light and never tarnish.



Figure 13.6 *A round brilliant-cut diamond. Its faces are angled so that most of the light entering the top is totally reflected inside and sent back out through the top.*
Photo: Petragems, CC BY-SA 4.0

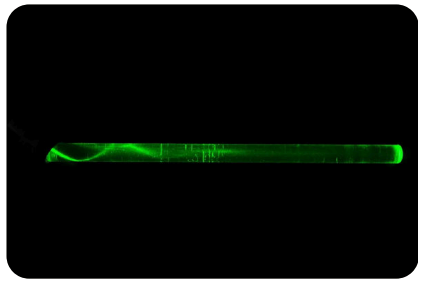


Figure 13.7 *A large model of an optical fiber: green laser light sent into a clear rod zigzags along it, totally reflected each time it meets the side, and comes out only at the far end.*
Photo: AZToshkov, CC BY-SA 3.0

Mirages. On a hot day, the road ahead seems to shine like water. Air near the hot road is hotter and optically rarer than the air above it. Light from the sky coming down at a shallow slant is bent more and more away from the normal as it passes through warmer and warmer layers, until, close to the road, it is turned back upward. Your eye sees the sky's light coming up from the road and takes it for a pool of water. (Strictly, the light is bent gradually all the way round, with no single surface where it reflects; textbooks describe the last turn as total internal reflection.)

WORKED EXAMPLE

Does it get out?

Light inside a glass block ($\mu = 1.5$) meets the glass–air surface at 45°. Does it get out? What would happen to light inside water ($\mu = 1.33$) meeting a water–air surface at 45°?

Answer

Glass: the critical angle is 41.8°, and 45° is more than that, so the light is **totally reflected**. Check with Snell's law: $\sin r = 1.5 \times \sin 45^\circ = 1.06$, and no angle has a sine bigger than 1, so there is no refracted ray.

Water: the critical angle is 48.8°, and 45° is less than that, so the light **gets out**: $\sin r = 1.33 \times \sin 45^\circ = 0.940$, so $r = 70.1^\circ$, bent well away from the normal.

Check 3

Total internal reflection can happen when light goes from:

a air into water
b air into glass
c glass into water
d water into glass

Check 4

The refractive index of a material is 2. What is its critical angle?

a 60°
b 30°
c 45°
d 2°

Total internal reflection around us

The sparkle of a diamond. A diamond's critical angle is only 24.4°, so light that enters it is very likely to meet its next face at more than that and be trapped. A diamond cutter cuts the stone with dozens of flat faces, angled so that light entering the top bounces from face to face inside and comes out of the top again, toward the viewer, instead of leaking out of the back. Glass, with a critical angle of 41.8°, lets far more light leak out at the back, so a glass copy cut the same way looks dull beside it. Diamond also splits white light into colors strongly, so the light comes out as flashes of color, the stone's "fire".

Optical fibers. An optical fiber is a glass thread about as thick as a human hair. Its middle, the **core**, has a slightly higher refractive index than the glass around it, the **cladding** (for example 1.48 and 1.46). Light sent into one end meets the core's edge at a big angle of incidence, more than the critical angle (here 80.6°), so it is totally reflected, every time, so it makes the best mirror there is.

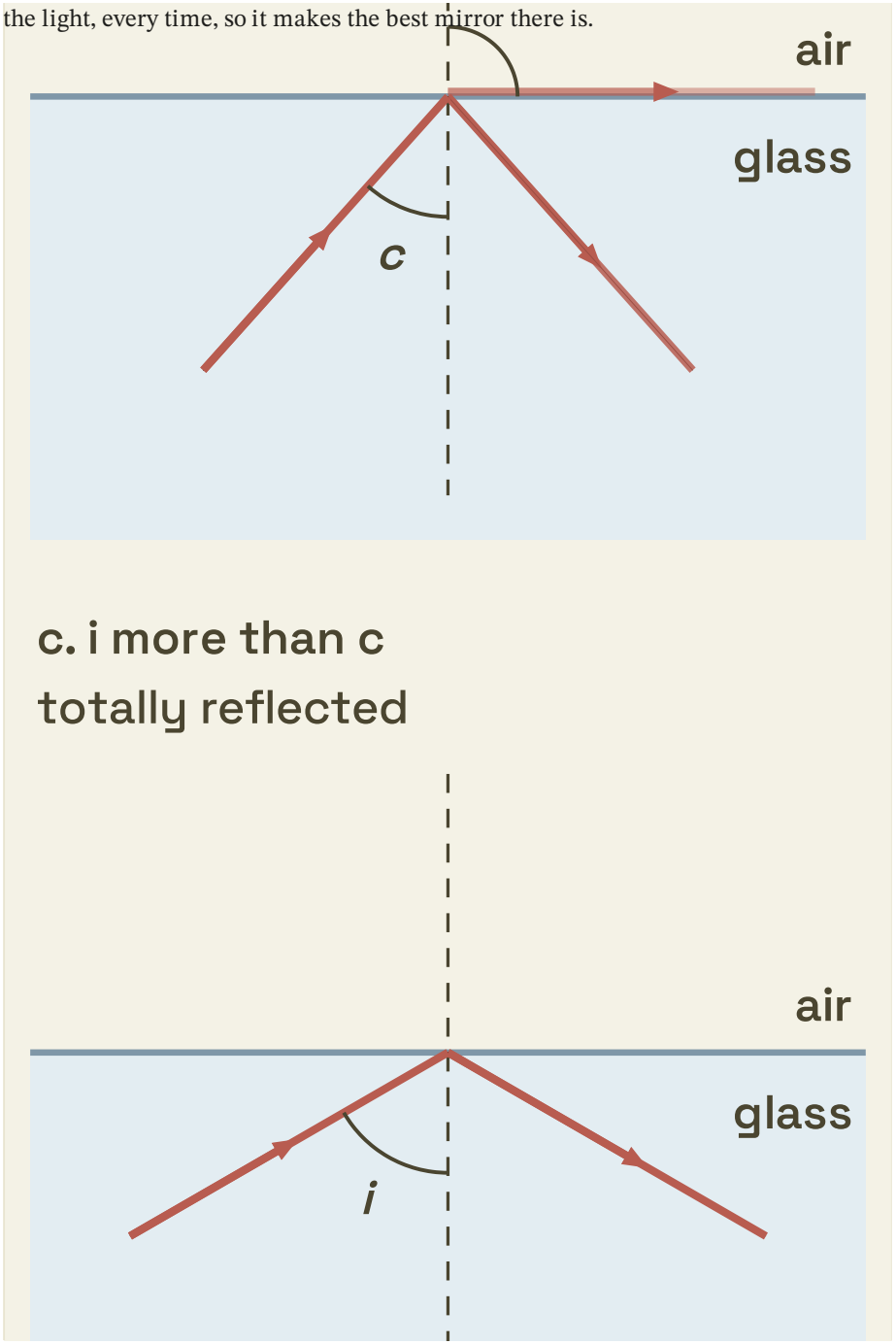


Figure 13.4 *Light inside glass (μ = 1.5) meeting the surface with air. a. At 30°, less than the critical angle, most of the light is refracted out, bent away from the normal; a little is reflected. b. At the critical angle, 41.8°, the refracted ray skims along the surface. c. At 60°, more than the critical angle, no light gets out: all of it is reflected back into the glass.*

CAREFUL

What really happens near the critical angle

Exam diagrams draw a bright ray skimming along the surface at the critical angle. In fact, as i gets closer to c, less and less light gets out and more and more is reflected; at the critical angle itself the light getting out has faded to nothing. Draw the skimming ray in an exam, as asked, but know that the change from refraction to total reflection is smooth, not a sudden switch.



Figure 13.5 *A green laser beam shone in at the end of a test tube full of water. Each time it meets the side, the angle of incidence is more than the critical angle, so it is totally reflected, all the way along the tube.*
Photo: Tanver Hossain Refat, CC BY 4.0

Worked examples

WORKED EXAMPLE

The critical angle of water and of diamond

Find the critical angles of water (μ = 1.33) and of diamond (μ = 2.42).

Answer

$$\sin c_{\text{water}} = \frac{1}{1.33} = 0.752 \qquad c = \sin^{-1}(0.752) = 48.8^\circ$$

$$\sin c_{\text{diamond}} = \frac{1}{2.42} = 0.413 \qquad c = \sin^{-1}(0.413) = 24.4^\circ$$

Water: 48.8°. Diamond: 24.4°, half as much.

WORKED EXAMPLE

The refractive index from the critical angle

The critical angle of a transparent plastic is 40°. Find its refractive index, and the speed of light in it.

Answer

$$\mu = \frac{1}{\sin c} = \frac{1}{\sin 40^\circ} = \frac{1}{0.643} = 1.56$$

$$v = \frac{c}{\mu} = \frac{3 \times 10^8}{1.56} = 1.93 \times 10^8 \text{ m/s}$$

μ = 1.56, and light travels at 1.93×10^8 m/s in it. (Careful: here the letter c is used for two things, the critical angle and the speed of light. Keep them apart by writing the units.)

WORKED EXAMPLE

Glass under water

A glass block (μ = 1.5) lies at the bottom of a tank of water (μ = 1.33). Find the critical angle for light going from the glass into the water.

Answer

$$\sin c = \frac{\mu_{\text{water}}}{\mu_{\text{glass}}} = \frac{1.33}{1.5} = 0.887 \qquad c = \sin^{-1}(0.887) = 62.5^\circ$$

62.5°, much more than glass's 41.8° in air. With water outside the glass instead of air, light gets out of the glass at many more angles, which is why a glass marble in water is harder to see than in air.