Unit 13

Answers: Light

Quick checks

1 b Air is optically rarer than water, so light going from water into air speeds up and bends **away** from the normal: $r > i$. (It is about 42°.) It would go straight on only at $i = 0^\circ$.

2 b $v = \frac{c}{\mu} = \frac{3\times10^8}{1.25} = 2.4 \times 10^8$ m/s. Light is always slower in a material than in a vacuum, so the answer must be less than 3×10^8 m/s: 3.75×10^8 m/s comes from multiplying instead of dividing.

3 c The light must go from an optically denser medium into a rarer one. Glass (1.5) is denser than water (1.33), so light in glass meeting water can be totally reflected, beyond 62.5°. Going into a denser medium, light can always get in.

4 b $\sin c = \frac{1}{\mu} = \frac{1}{2} = 0.5$, so $c = 30^\circ$.

5 b The bending at the second face exactly undoes the bending at the first, because the faces are parallel, so $e = i$: the emergent ray is parallel to the incident ray. But it has traveled sideways inside the glass, so it is shifted (the lateral shift), not on the same line.

6 b Apparent depth = 16 ÷ (4/3) = 12 cm, so it seems raised by 16 − 12 = **4 cm**. 12 cm is the apparent depth, not the shift; 21.3 cm comes from multiplying by μ instead of dividing.

1 a. **b** Glass is optically denser than air: light slows down in it and bends toward the normal, so $r < i$.

1 b. **d** $\mu = \frac{c}{v}$ is one speed divided by another (or one sine divided by another), so the units cancel.

1 c. **c** This is the definition of the critical angle. For any bigger angle of incidence, total internal reflection takes place.

1 d. **c** 50° is more than the critical angle and the light is going from a denser into a rarer medium, so both conditions for total internal reflection are met.

1 e. **b** At the critical angle, $\mu \sin c = \sin 90^\circ = 1$, so $\sin c = \frac{1}{\mu}$ and $\mu = \frac{1}{\sin c}$.

1 f. **c** Apparent depth = real depth ÷ μ = 2 ÷ (4/3) = 1.5 m. Multiplying by μ instead gives 2.67 m, and 0.67 m is 2 ÷ 3, a slip with the fraction.

1 g. **b** Light inside the core meets its edge at more than the critical angle and is totally reflected, again and again, so it follows the fiber. There is no silver: the cladding of slightly lower refractive index is what makes it work.

1 h. **b** The faces of the slab are parallel, so the bending at the second face undoes the bending at the first: $e = i$, and the emergent ray is parallel to the incident ray.

2 a. Light from the bottom of the pond passes from water (optically denser) into air (rarer), so it bends away from the normal at the surface. The eye traces the light back in a straight line and sees the bottom where the light seems to come from, higher than it really is. So the apparent depth is less than the real depth (about three-quarters of it, looking straight down), and the pond looks shallower.

2 b. The critical angle of diamond is only 24.4°, against 41.8° for glass. Light entering a diamond meets its inner faces at more than 24.4° most of the time, so it is totally reflected again and again and comes back out through the top, toward the viewer. In glass, more of the light meets the faces at less than 41.8° and leaks out at the back. Diamond also spreads white light into colors more strongly. So a diamond sends far more light, in flashes of color, to the eye.

2 c. When light meets the surface along the normal, the whole of its front enters the glass at the same moment, so it all slows down together and nothing makes it turn. By Snell's law, $i = 0^\circ$ gives $\sin r = \frac{\sin 0^\circ}{\mu} = 0$, so $r = 0^\circ$: it goes straight on, although its speed still changes.

2 d. Total internal reflection needs light going from an optically denser into a rarer medium. Air is rarer than water. Going from air into water, light bends toward the normal, so the angle of refraction is always smaller than the angle of incidence and never reaches 90°: there is no critical angle, and light can always get in.

2 e. Light meets the inside surface of the tube's glass, where the glass has air behind it, at more than the critical angle, so it is totally reflected to the eye and the tube shines like a mirror. Filled with water, the tube has water behind the glass instead of air; the critical angle from glass into water is about 62.5°, so the light passes into the tube instead of being totally reflected, and the silver look disappears.

3 i.

Angle of incidence	Critical angle
The angle between the incident ray and the normal, whatever its size.	The particular angle of incidence, in the denser medium, for which the angle of refraction is 90°.
Can be anything from 0° to 90°, set by where the light comes from.	Fixed for a given pair of media: 41.8° for glass and air, 48.8° for water and air.
Defined for light going either way, from rarer to denser or denser to rarer.	Exists only for light going from a denser into a rarer medium.

3 ii.

Reflection from a mirror	Total internal reflection
Happens at a polished or silvered surface.	Happens at the boundary of a denser medium with a rarer one, inside the denser medium.
Happens at any angle of incidence.	Happens only when the angle of incidence is greater than the critical angle.
Some light is always absorbed, so the reflected light is less bright.	All the light is reflected; none is lost.
A silvered mirror can tarnish and get dull.	Never tarnishes; it depends only on the two media.

3 iii.

Real depth	Apparent depth
The actual distance of an object below the surface of a medium.	The distance below the surface at which the object appears to be, where its image is.
Measured directly, for example with a stick or a scale.	Found by looking: for water seen from above it is about three-quarters of the real depth.
Bigger of the two for an object in a denser medium seen from air.	Smaller: real depth ÷ apparent depth = μ, looking straight down.

4 a. **Refraction of light** is the change in the direction of light when it passes at a slant from one transparent medium into another, because its speed changes.
Laws of refraction:

1. The incident ray, the refracted ray and the normal at the point of incidence all lie in the same plane.
2. **Snell's law:** for a given pair of media and a given color of light, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant, the refractive index of the second medium with respect to the first: $\frac{\sin i}{\sin r} = \mu$.

4 b. The **absolute refractive index** of a medium is its refractive index with respect to a vacuum (taken as the same as air). Refraction happens because light changes speed, and the amount of bending is set by how much: $\mu = \frac{\sin i}{\sin r} = \frac{c}{v}$, where c is the speed of light in a vacuum and v its speed in the medium. For example, for glass, $\frac{3\times10^8}{2\times10^8} = 1.5$, the same value as $\frac{\sin i}{\sin r}$ measured in the pin experiment.

4 c. **Total internal reflection** is the reflection of all the light back into a denser medium when it meets the boundary with a rarer medium at an angle of incidence greater than the critical angle.
Conditions:

1. The light must travel from an optically denser medium into an optically rarer medium.
2. The angle of incidence must be greater than the critical angle.

Experiment: Fill a clear tank with water and add a few drops of milk. In a darkened room, shine a laser pointer up through the side of the tank so that the beam meets the water surface from below. With the beam steep, it comes out through the surface, bent away from the normal, and a weak beam is reflected. Lower the pointer so that the beam travels flatter and the angle of incidence increases: the emerging beam moves closer to the surface and gets fainter. Beyond about 49°, the critical angle of water, no beam comes out: the whole beam is reflected back down into the water. This is total internal reflection.

4 d. Let light pass from a medium of refractive index μ into air (refractive index 1). By Snell's law, $\mu \sin i = 1 \times \sin r$. At the critical angle, $i = c$ and $r = 90^\circ$, and $\sin 90^\circ = 1$:

$$\mu \sin c = 1 \qquad \sin c = \frac{1}{\mu} \qquad \mu = \frac{1}{\sin c}$$

So the higher the refractive index, the smaller the critical angle. For example, diamond ($\mu = 2.42$) has $c = 24.4°$, glass ($\mu = 1.5$) has $41.8°$.

4 e. Place the slab on a sheet of paper on a drawing board and draw its outline ABCD. Draw a normal at a point E on AB, and an incident ray at an angle i (say 40°) to it. Stick two pins P_1 and P_2 upright on the incident ray. Put the slab back on its outline, and looking through face DC, stick pins P_3 and P_4 in line with the images of P_1 and P_2. Remove the slab, draw the line P_3P_4 to meet DC at F, and join EF, the refracted ray. Measure the angle of refraction r between EF and the normal at E. Then $\mu = \frac{\sin i}{\sin r}$. Repeat for several angles of incidence and take the average. The diagram is Figure 13.9.

Precautions: the pins upright and well apart; angles of incidence between 30° and 60°; look at the feet of the pins; use a sharp pencil.

4 f. Let O be the object at the bottom and A the point on the surface straight above it. A ray along OA goes straight up. A ray along OB meets the surface at B at angle of incidence i, and leaves bent away from the normal at angle r; traced back, it meets OA at I, the image. Then AO is the real depth and AI the apparent depth.

The angle AOB = i and the angle AIB = r. From the triangles OAB and IAB, $\tan i = \frac{AB}{AO}$ and $\tan r = \frac{AB}{AI}$.

For light going from water into air, $\mu = \frac{\sin r}{\sin i}$. Looking straight down, the angles are very small, so $\sin i \approx \tan i$ and $\sin r \approx \tan r$. Therefore

$$\mu = \frac{\tan r}{\tan i} = \frac{AB/AI}{AB/AO} = \frac{AO}{AI} =$$

4 g. **Uses (any two):** optical fibers for telephone and internet; endoscopes for seeing inside the body; right-angled prisms in binoculars and periscopes; cutting diamonds so that they sparkle.

How an optical fiber works: an optical fiber is a fine glass thread with a core of slightly higher refractive index than the cladding around it. Light sent into one end meets the boundary between core and cladding at an angle of incidence greater than the critical angle, so it is totally reflected. It bounces along the core from side to side, following the fiber round bends, and comes out at the other end with very little light lost, even after many kilometers.

5 a.

$$\mu = \frac{\sin 45^\circ}{\sin 28^\circ} = \frac{0.707}{0.469} = 1.51$$

The refractive index is about 1.51.

5 b.

$$\mu = \frac{3 \times 10^8}{2.25 \times 10^8} = 1.33$$

$$\sin c = \frac{1}{1.33} = 0.75 \qquad c = \sin^{-1}(0.75) = 48.6^\circ$$

μ = 1.33 and the critical angle is 48.6°. (The liquid is probably water.)

5 c.

$$\sin r = \frac{\sin 60^\circ}{1.47} = \frac{0.866}{1.47} = 0.589 \qquad r = 36.1^\circ$$

About 36°.

5 d.

$$\mu = \frac{1}{\sin 35^\circ} = \frac{1}{0.574} = 1.74$$

$$v = \frac{3 \times 10^8}{1.74} = 1.72 \times 10^8$$

μ = 1.74, and light travels at 1.72×10^8 m/s.

5 e.

$$= \frac{1.2}{4/3} = 0.9$$

0.9 m.

5 f. Apparent depth $= 3.6 - 1.2 = 2.4$ cm.

$$\mu = \frac{3.6}{2.4} = 1.5$$

μ = 1.5.

5 g. Real depth $= 4.90 - 1.12 = 3.78$ cm; apparent depth $= 4.90 - 2.08 = 2.82$ cm.

$$\mu = \frac{3.78}{2.82} = 1.34$$

The refractive index is about 1.34 (it could be water).

5 h.

$$_g\mu_w = \frac{v}{v} = \frac{2 \times 10^8}{2.25 \times 10^8} = 0.889$$

Yes: light is slower in glass, so glass is the optically denser medium, and light going from glass into water can be totally reflected.

$$\sin c = 0.889 \qquad c = \sin^{-1}(0.889) = 62.7^\circ$$

0.889; yes, with a critical angle of about 63°.

5 i. Light from the lamp meets the surface at a larger angle of incidence the farther it is from the point A straight above the lamp. Up to the critical angle, 48.8°, it gets out; beyond that, it is totally reflected back into the pool. So light escapes only through a circle centered on A, whose edge is where the angle of incidence equals the critical angle. With depth d, the radius is

$$R = d \tan c = 2 \times \tan 48.8^\circ = 2 \times 1.14 = 2.28$$

A circle about 2.3 m in radius. (Turned round, this is Snell's window: a swimmer at the bottom sees the whole sky inside that same circle.)