Unit 11

Answers: Force

Quick checks

1 c Weight is a force, and it always points downward, so it has a direction: a vector. Mass, time and speed have magnitude only. (The bus's velocity would be a vector; its speed is not.)

2 d The resultant is at most $6 + 4 = 10$ N (same direction) and at least $6 - 4 = 2$ N (opposite directions). 12 N is more than the largest possible.

3 b The two velocities are at right angles, so $v = \sqrt{3^2 + 4^2} = 5$ m/s. 7 m/s would be right only if the boat went with the current, and 1 m/s only if against it.

4 c Taking "away from the wall" as positive, the velocity goes from −5 m/s to +5 m/s: a change of $5 - (-5) = 10$ m/s. The speed is the same, but the direction has reversed, and that is a change of velocity.

5 b The distance is the area of the triangle under the line: $\frac{1}{2} \times 4 \times 20 = 40$ m. 80 m is the rectangle, forgetting the half; 5 comes from dividing 20 by 4, which gives the slope (a retardation of 5 m/s²), not the area.

6 c A flat line means the velocity doesn't change: zero slope, zero acceleration. The area is a rectangle, $8 \times 5 = 40$ m. ($8 \div 5 = 1.6$ is a slope you might get by mistake, dividing instead of multiplying.)

7 c $M = F \times d = 20 \times 0.25 = 5$ N m. Moments are in newton meters, not joules; 80 would come from dividing 20 by 0.25.

8 b Moment = force × perpendicular distance. Near the hinges the distance is small, so the same force gives a small moment, and a much bigger force is needed for the same turning effect.

9 c Opening a tap, the thumb and fingers push with equal and opposite forces on opposite sides of the handle: a couple. Each of the others is a single force turning a body about a pivot.

10 b Moment of a couple = one force × the distance between them = $15 \times 0.40 = 6$ N m. 3 N m would use half the diameter (the radius) for only one hand; 600 forgets to change centimeters into meters.

11 b The truck's velocity is zero, so its momentum is $5{,}000 \times 0 = 0$. The runner's is $60 \times 5 = 300$ kg m/s. A heavy body at rest has a lot of inertia but no momentum.

12 b The change in momentum is fixed: the ball arrives with its momentum and must end with none. Drawing her hands back makes the stop take longer, and since $F = \frac{\text{–}}{t}$, a longer time means a smaller force.

13 **b** Before firing, gun and bullet are at rest: total momentum zero. The explosion pushes them apart with equal and opposite forces (Newton's third law), so their momenta after are equal and opposite, and still add to zero.

14 **b** Momentum before = $1 \times 4 = 4$ kg m/s. Afterward the 2 kg pair has the same momentum: $2 \times v = 4$, so $v = 2$ m/s. The moving mass doubled, so the velocity halved.

1 a. **b** Displacement, acceleration and weight all have a direction. Each of the other groups has a scalar in it: mass, speed and time, work and electric current.

1 b. **c** At right angles, $R = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$ N. 17 N would be the resultant if they acted in the same direction, 7 N if they acted in opposite directions.

1 c. **c** Slope = change in velocity ÷ time taken = acceleration. The area under the graph gives the distance.

1 d. **b** Moment = force × perpendicular distance, so its unit is newton × meter, N m. It is not the joule, the unit of work, even though both are a newton times a meter.

1 e. **b** The two forces of a couple are equal and opposite, so their resultant is zero: no movement from place to place. Because they don't act along the same line, they do have a moment, and turn the body.

1 f. **c** By Newton's second law, $F = \dfrac{mv - mu}{t}$: the force equals the rate of change of momentum.

1 g. **c** $p = mv = 2 \times 3 = 6$ kg m/s.

2 a. Distance is the total length of the path traveled; it has magnitude only, and a number with a unit (5 km) describes it fully. Displacement is the straight-line change in position **in a particular direction**, so it needs both magnitude and direction (5 km to the north), and displacements add by the rules of vectors. So distance is a scalar and displacement a vector.

2 b. The moment of a force = force × perpendicular distance from the pivot. The handle is as far as possible from the hinges, so the perpendicular distance is largest, and a small force gives a large enough moment to open the door easily.

2 c. To loosen the nut a certain moment is needed, and moment = force × perpendicular distance. With a longer wrench the force acts farther from the nut, so a smaller force gives the same moment. Doubling the length of the wrench halves the force needed.

2 d. The ball must lose all its momentum, so the change in momentum is fixed. By Newton's second law, force = change in momentum ÷ time. Drawing her hands back makes the catch last longer, so the force on her hands is smaller and they hurt less.

2 e. Before firing, the gun and bullet are at rest, so their total momentum is zero. When the bullet is fired forward it gains forward momentum; by the law of conservation of linear momentum the total must still be zero, so the gun gains an equal momentum backward and recoils. Its velocity is small because its mass is large.

2 f. The person and the boat are at rest, with zero total momentum. When the person steps forward onto the bank, the push of their feet gives them forward momentum and gives the boat an equal backward momentum, so the total stays zero. The boat therefore moves backward, away from the bank.

2 g. Distance = velocity × time. On a velocity–time graph the height of the region is a velocity and its width is a time, so its area is velocity × time, a distance. For a steady velocity it is a rectangle, velocity × time; for a changing velocity the area can be cut into thin strips, each of which is (velocity at that time) × (short time), and their sum is the whole distance.

3 i.

Scalar quantity	Vector quantity
Has magnitude only.	Has both magnitude and direction.
Added like ordinary numbers.	Added by the triangle or parallelogram law.
Written as a number with a unit.	Drawn as an arrow, written in bold or with an arrow over it.
Examples: mass, distance, speed, time, energy.	Examples: displacement, velocity, acceleration, force, momentum.

3 ii.

Moment of a force	Couple
The turning effect of a single force about a pivot.	A pair of equal, opposite, parallel forces not in the same line.
Moment = force × perpendicular distance from the pivot.	Moment = one force × the perpendicular distance between the two forces.
Its value depends on the point it is taken about.	Its moment is the same about every point.
The force can also move the body along, unless a pivot holds it.	The forces add up to zero, so it only turns the body.
Example: opening a door, a see-saw.	Example: a steering wheel, a water tap.

3 iii.

Uniform velocity	Uniform acceleration
A straight line parallel to the time axis.	A straight line sloping up (or down, for uniform retardation).
Its slope, the acceleration, is zero.	Its slope is the same everywhere: a constant acceleration.
The area under it is a rectangle.	The area under it is a triangle or a trapezium.

4 a. The **resultant** of two or more vectors is the single vector that has the same effect as all of them acting together.

Triangle law: if two vectors are represented in magnitude and direction by two sides of a triangle taken in order, their resultant is represented in magnitude and direction by the third side, taken from the tail of the first to the head of the second.

Parallelogram law: if two vectors acting at a point are represented in magnitude and direction by two adjacent sides of a parallelogram drawn from that point, their resultant is represented by the diagonal of the parallelogram passing through that point.

4 b. To find $\vec{A} - \vec{B}$, reverse **B** to get **−B** (same size, opposite direction) and add it to **A** by the triangle law: $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$.
Velocity is a vector, so a change in its direction is a change in velocity, even at the same speed. Taking "away from the wall" as positive, the velocity changes from -6 m/s to $+6$ m/s: change $= 6 - (-6) =$ **12 m/s away from the wall**.

4 c.

- **Acceleration** is the slope of the line: $a = \dfrac{v_2 - v_1}{t_2 - t_1}$ between two points on a straight part. A rising line gives a positive acceleration, a falling line a retardation, a flat line zero.
- **Distance** is the area between the line and the time axis, worked out as rectangles (uniform velocity), triangles and trapeziums (uniform acceleration).
- **Average velocity** is the total area (total displacement) divided by the total time.

4 d. For a body with uniform acceleration a, its velocity–time graph is a straight line from u at time 0 to v at time t.

First equation. The slope is the acceleration: $a = \dfrac{v - u}{t}$, so $v = u + at$.

Second equation. The distance s is the area under the line. It is a rectangle of height u and width t, area ut, with a triangle on top of base t and height $v - u = at$, area $\frac{1}{2} \times t \times at = \frac{1}{2}at^2$. So

$$s = ut + \tfrac{1}{2}at^2$$

4 e. The **moment of a force** about a point is the product of the force and the perpendicular distance from the point to the force's line of action, $M = F \times d$, measured in N m.
It depends on **the size of the force** and **the perpendicular distance** of its line of action from the pivot.
Examples (any two): the long handle of a wrench or lug wrench; a door handle fixed far from the hinges; the long handle of a hand pump; long-handled shears for thick branches; a wheelbarrow lifted at the ends of its handles.

4 f. **Principle of moments:** when a body is in equilibrium, the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about the same point.
Anticlockwise moment = 3 × 20 = 60 N cm. For balance, 2 × d = 60, so d = 30 cm.
The 2 N weight must hang 30 cm to the right of the pivot (at the 80 cm mark).

4 g. A **couple** is a pair of equal and opposite parallel forces whose lines of action are different.
Let the forces be F each, with a perpendicular distance d between them. Taking moments about the point midway between them, each force is $\frac{d}{2}$ away and both turn the body the same way:

$$= F \times \frac{d}{2} + F \times \frac{d}{2} = F \times d$$

Examples (any four): turning a steering wheel; opening a water tap; turning a key in a lock; unscrewing a bottle cap or jar lid; turning a screwdriver; winding a watch; turning a lug wrench with both hands; the Earth's field turning a compass needle.

4 h. **Linear momentum** is the product of a body's mass and velocity, $p = mv$, a vector in kg m/s.
Let a force F act on a body of mass m for time t, changing its velocity from u to v. The change in momentum is $mv - mu$, and its rate of change is $\dfrac{mv - mu}{t}$. By Newton's second law, the force is proportional to the rate of change of momentum:

$$F \propto \frac{m(v-u)}{t} = ma \quad \Rightarrow \quad F = kma$$

The newton is the force that gives 1 kg an acceleration of 1 m/s², so $k = 1$ and $F = ma$.

4 i. **Law:** when no external force acts on a system of bodies, their total momentum stays the same.

Proof: two bodies of masses m_1 and m_2, with velocities u_1 and u_2, collide for a time t and move on at v_1 and v_2. By Newton's third law, the force of B on A equals minus the force of A on B: $F_{AB} = -F_{BA}$. By the second law, each force is the rate of change of momentum of the body it acts on:

$$\frac{m_1 v_1 - m_1 u_1}{t} = -\frac{m_2 v_2 - m_2 u_2}{t}$$

Multiplying by t and rearranging:

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

The total momentum before the collision equals the total momentum after it.

4 j. A rocket burns fuel and pushes the hot gases out of its nozzle backward at a very high speed. The gases gain a large backward momentum. By the law of conservation of linear momentum (or Newton's third law: the rocket pushes the gas back, the gas pushes the rocket forward), the rocket gains an equal forward momentum. It pushes against its own exhaust, not against air, so it works in empty space.

5 a.

$$R = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$$

$$\tan\theta = \frac{8}{6} = 1.33 \quad \Rightarrow \quad \theta = 53.1^\circ$$

10 N, at 53.1° to the 6 N force.

5 b. i. $6 + 8 =$ **14 N**, in their common direction.

ii. $8 - 6 =$ **2 N**, in the direction of the 8 N force.

5 c.

$$v = \sqrt{240^2 + 70^2} = \sqrt{57{,}600 + 4{,}900} = \sqrt{62{,}500} = 250$$

$$\tan\theta = \frac{70}{240} = 0.292 \quad \Rightarrow \quad \theta = 16.3^\circ$$

250 km/h, at 16.3° east of north.

5 d. The graph rises from 0 to 20 m/s (0 to 8 s), stays at 20 m/s (8 to 20 s), and falls to 0 (20 to 24 s).

i. $a = \dfrac{20 - 0}{8} = 2.5\ ^2$

ii. $a = \dfrac{0 - 20}{4} = -5\ ^2$, a retardation of **5 m/s²**.

iii. Distance $= \frac{1}{2} \times 8 \times 20 + 12 \times 20 + \frac{1}{2} \times 4 \times 20 = 80 + 240 + 40 =$ **360 m**.

iv. Average velocity $= 360 \div 24 =$ **15 m/s**.

5 e. Slope: $a = \dfrac{4 - 10}{3} = -2\ ^2$, a **retardation of 2 m/s²**.
Area of the trapezium: $\frac{1}{2} \times (10 + 4) \times 3 =$ **21 m**.

5 f. From 0 to 6 s the velocity rises uniformly (acceleration $3 \div 6 = 0.5$ m/s²); from 6 to 10 s it stays 3 m/s.
Distance = triangle + rectangle = $\frac{1}{2} \times 6 \times 3 + 4 \times 3 = 9 + 12 =$ **21 m**.
Average velocity = $21 \div 10 =$ **2.1 m/s**.

5 g. $M = F \times d = 40 \times 0.25 =$ **10 N m**.
At 10 cm: $F = \dfrac{10}{0.10} =$ **100 N**.

5 h. By the principle of moments, $40g \times 2 = 32g \times d$, so

$$d = \frac{40 \times 2}{32} = 2.5$$

Rabina must sit 2.5 m from the pivot.

5 i. Moment = $F \times d = 25 \times 0.08 =$ **2 N m**.

5 j. $u = 72$ km/h $= \dfrac{72 \times 1000}{3600} = 20$ m/s, $v = 0$.
Change in momentum = $mv - mu = 0 - 1500 \times 20 = -30{,}000$ kg m/s (a decrease of 30,000 kg m/s).

$$F = \frac{mv - mu}{t} = \frac{-30{,}000}{4} = -7{,}500$$

The momentum falls by 30,000 kg m/s, and the braking force is 7,500 N, against the motion.

5 k. $0 = 0.020 \times 12 + 0.8 \times v$, so $v = -\dfrac{0.24}{0.8} = -0.3$ m/s.
The cannon recoils at 0.3 m/s, opposite to the ball.

5 l. Before: $60 \times 4 + 40 \times 0 = 240$ kg m/s. After: $(60 + 40)$ v = 240, so v = **2.4 m/s**, in Bishal's direction.

5 m. Total momentum before = $2 \times 3 + 1 \times (-2) = 6 - 2 = 4$ kg m/s.
After: $(2 + 1)$ v = 4, so v = 1.33 m/s.
They move together at 1.33 m/s, in the direction the 2 kg trolley was moving.

5 n. Total momentum before = 0. After: $2 \times 5 + 100 \times v = 0$, so $v = -0.1$ m/s: **she moves toward the station at 0.1 m/s**, opposite to the tool.
Time = 10 m ÷ 0.1 m/s = **100 s**, under two minutes.
Waving her arms moves parts of her body, but the total momentum of her and her suit stays zero, since nothing is thrown away and there is no air to push against: when her arms stop, she stops. To move, she must throw some mass away, which is exactly how a rocket works.