Unit 11 · Force

STUDY NOTES

Key terms

Scalar: magnitude only (mass, time, speed, energy, current, pressure).

Vector: magnitude and direction, adds by vector addition (displacement, velocity, acceleration, force, weight, momentum).

Resultant: single vector with the same effect as two or more together.

Line of action: straight line along which a force acts, through its point of application.

Pivot: fixed point or line a body turns about (fulcrum, axis of rotation).

Moment (torque): turning effect of a force.

Couple: pair of equal, opposite, parallel forces whose lines of action are not the same.

Arm of the couple: perpendicular distance between the two lines of action.

Retardation: negative acceleration, a line sloping down on a v–t graph.

Linear momentum: mass × velocity; a vector, kg m/s.

Impulse: force × time it acts = change in momentum.

System: the group of bodies looked at; forces among them **internal**, from outside **external**.

Formulas

$R = \sqrt{A^2 + B^2}$, $\tan\theta = B/A$	θ angle of R with A **Use:** two vectors at right angles
$a = \dfrac{v_2 - v_1}{t_2 - t_1}$	slope of a v–t line (m/s^2) **Use:** acceleration from a graph
$v = u + at$	from the slope **Use:** speed after a time
$s = \dfrac{(u + v)}{2} \times t$	trapezium under the line **Use:** distance, one uniform stretch
$s = ut + \frac{1}{2}at^2$	rectangle + triangle **Use:** distance with a and t
$v^2 = u^2 + 2as$	i. and (ii) with t removed **Use:** no time given
average velocity = total area ÷ total time	area = displacement **Use:** whole journey
$M = F \times d$	d perpendicular distance (m); N m **Use:** moment of a force
moment of couple $= F \times d$	one force × arm **Use:** same about every point
$p = mv$	m kg, v m/s **Use:** momentum
$F = \dfrac{mv - mu}{t} = ma$	rate of change of momentum **Use:** $Ft = mv - mu$
$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$	no external force **Use:** collisions, recoil
$v_g = -\dfrac{m_bv_b}{m_g}$	gun g, bullet b; minus = backward **Use:** recoil

Vectors

- drawn as an **arrow**
  - points in the direction
  - length = magnitude, to a **scale** (1 cm = 10 N: 40 N east → 4 cm east)
  - **tail** at the start, **head** at the point
- written bold **F** or $\vec{F}$; magnitude alone F or $|\vec{F}|$
- **equal vectors**: same magnitude and direction, anywhere
- **negative −A**: same magnitude, opposite way
- distance vs displacement: Kathmandu–Pokhara and back, about 200 km each way → distance 400 km, displacement **zero**

Three that fool people:

- electric current: scalar (currents at a junction add as numbers, whatever the angle)
- pressure: scalar (pushes equally every way at a point)
- weight: vector; mass: scalar

Adding:

- same direction: $R = A + B$ (40 N + 30 N → 70 N)

- opposite: $R = A - B$, in the larger's direction (1,500 N, 1,350 N → 150 N)
- so R is **largest** at same direction and **smallest** at opposite; at any other angle between the two (6 N, 4 N → 2 N to 10 N)
- right angles: Pythagoras (4 km E, 3 km N → 5 km at 36.9° N of E; distance 7 km)
- any other angle: **graphical method**, to scale head to tail, ruler and protractor; any number of vectors, first tail to last head

Triangle law: if two vectors are represented in magnitude and direction by the two sides of a triangle taken in order, their resultant is represented in magnitude and direction by the third side, from the tail of the first to the head of the second.

Parallelogram law: if two vectors acting at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from that point, their resultant is represented in magnitude and direction by the diagonal from the same point.

Subtracting: $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

- change in velocity = final − initial
- ball at 8 m/s, bounces back at 6 m/s: $6 - (-8) = 14$ m/s away from the wall, not 2
- bus 10 m/s E → 10 m/s N: speed same, velocity changes by 14.1 m/s NW

River: crossing time depends only on the velocity across (200 m ÷ 4 m/s = 50 s; flow 3 m/s → 150 m downstream; resultant 5 m/s)

Rain: velocity relative to a cyclist = rain's − hers (6 m/s down, 4.5 m/s → 7.5 m/s at 36.9° to the vertical; tilt the umbrella forward)

Velocity–time graph

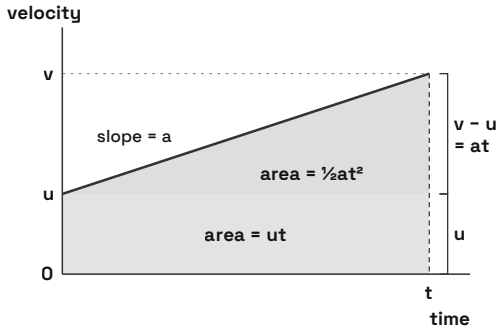
Plotting:

- time across (the quantity we choose), velocity up
- easy scales filling the paper
- label quantity and unit
- crosses or dots
- ruler for steady change, smooth curve otherwise

Line	Means
up	positive slope, speeding up
flat	zero slope, uniform velocity
down	negative slope, retardation
steeper	greater acceleration
curved	slope at a point = slope of its **tangent**

Shape under the line	Area
rectangle (uniform velocity)	width × height
triangle (from or to rest)	½ × base × height
trapezium (between two velocities)	½ × (sum of parallel sides) × width

- area = velocity × time = **distance** (strictly displacement); read s and m/s from the axes
- **below the axis**: displacement in the negative direction
- ball thrown up at 20 m/s ($g \approx 10$ m/s^2): line from +20 to −20, crossing at 2 s; areas +20 m, −20 m → displacement 0, distance 40 m
- straight all the way → **acceleration at the top still 10 m/s^2 down**; zero velocity ≠ zero acceleration



Slope = a; area = rectangle ut + triangle $\frac{1}{2}at^2 = s$. Redraw it for the derivations.

Deriving:

- slope: $a = (v - u)/t \rightarrow v = u + at$
- trapezium: $s = (u + v)t/2$
- rectangle + triangle (height $v - u = at$): $s = ut + \frac{1}{2} \times t \times at = ut + \frac{1}{2}at^2$
- $t = (v - u)/a$ into the trapezium: $s = (v^2 - u^2)/2a \rightarrow v^2 = u^2 + 2as$

Practical 22, microbus:

- OA 0–15 s, 0.8 m/s^2, 90 m
- AB 15–30 s at 12 m/s, 180 m
- BC 30–40 s, −1.2 m/s^2, 60 m
- Total 330 m; average velocity 8.25 m/s

Moments

Moment about a point = force × perpendicular distance from the point to the line of action.

- larger by a **bigger force** or a **bigger perpendicular distance** (usually easier)
- 1 N m: 1 N acting 1 m from the pivot, at right angles
- sense matters: **clockwise** or **anticlockwise**; equal and opposite moments cancel
- change cm to m first: 50 N at 30 cm → 15 N m
- push at 30° to the handle: 150 N, 16 cm handle → perpendicular distance 8 cm → 12 N m, half
- uses of a long arm
  - door handles far from hinges
  - wrenches
  - hand pump

- tin snips and shears
- wheelbarrow
- bottle opener

- pedal: easiest with the crank horizontal; crank straight down → push through the axle, no moment

Principle of moments: when a body is in equilibrium, the sum of the clockwise moments about any point is equal to the sum of the anticlockwise moments about the same point.

- see-saw: 30 kg at 1.5 m balances 45 kg at 1.0 m; g cancels; heavier nearer the pivot

Couple

- forces add up to zero → no moving from its place, **only turns**
- about the midpoint: $F \times \frac{d}{2} + F \times \frac{d}{2} = F \times d$; about one hand: that force has no moment, other gives $F \times d$
- examples
  - steering wheel
  - bicycle handlebar with both hands
  - tap
  - key
  - bottle cap
  - jar lid
  - screwdriver
  - winding a watch
  - lug wrench with both hands
  - compass needle
  - coil of a motor
- lug wrench, 120 N each, 40 cm apart: 48 N m; one hand at one end, 20 cm from the nut: 24 N m, and it pushes sideways
- compass turns but doesn't slide: couple; along the field both forces in one line, moment zero

Moment of a force	Couple
one force about a pivot	two equal, opposite, parallel forces
depends on the point	same about every point
can also move the body (pivot holds it)	only turns
door, wrench on a nut	steering wheel, tap, key

Momentum and force

- body at rest: **no momentum**, however heavy (parked truck 0; 60 kg runner at 5 m/s, 300 kg m/s)
- straight road: pick a positive direction; opposite → negative velocity and momentum

Newton's second law: the rate of change of momentum of a body is directly proportional to the force applied on it, and takes place in the direction of the force.

- $F \propto m(v-u)/t = ma \rightarrow F = k\,ma$; newton: gives 1 kg 1 m/s^2, so $k = 1$
- units: N × s = kg m/s
- same change in momentum: big force, short time or small force, long time
- catch: 0.16 kg at 25 m/s, 4 kg m/s: 0.01 s → 400 N; 0.1 s → 40 N
- **longer stop, smaller force**
  - hands drawn back
  - foam mattress
  - sand pit
  - bent knees
  - straw or foam packing
  - helmet foam
  - airbags
  - crumple zones
- bounce > dead stop: 0.5 kg, −10 to +8 m/s: change 9 kg m/s, not 1; in 0.05 s, 180 N

Conservation of momentum

Law: when no external force acts on a system of bodies, the total momentum of the system stays the same.

Proof, two balls colliding for time t:

- third law: $F_{AB} = -F_{BA}$, lasting the same time
- second law: $F_{AB} = (m_1v_1 - m_1u_1)/t$; $F_{BA} = (m_2v_2 - m_2u_2)/t$
- substitute, multiply by t, rearrange: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
- holds for bouncing, sticking or breaking, with no outside force

Uses (total momentum zero before):

- **gun**: forward bullet, backward gun, slower as heavier; held to the shoulder so the mass is larger. 4 kg, 10 g at 400 m/s → 1 m/s back
- **boat**: step forward, boat back. 50 kg at 1.5 m/s, 150 kg → 0.5 m/s back
- **rocket, jet, balloon**: gas back, rocket forward; **no air needed**, works in space
- hose
- swimmer
- walking (Earth moves too little to notice)
- astronaut throwing something

Sticking: 2 kg at 3 m/s + 1 kg at rest → $3v = 6$, $v = 2$ m/s

Head-on: 3,000 kg E at 10 m/s, 1,000 kg W at 20 m/s: 10,000 kg m/s ÷ 4,000 kg → 2.5 m/s E; changes 7.5 m/s (truck) and 22.5 m/s (car)

COMMON MISTAKES

✗ A wrench or a door is a couple.
✓ One force; give a steering wheel, a tap or a key.

✗ A moment is in joules.
✓ N m; distance at right angles, nothing need move.

✗ Distance in a moment is along the handle whatever the push.
✓ Perpendicular distance to the line of action.

✗ Average velocity = average of the readings.
✓ Total area ÷ total time; $(u+v)/2$ only for one uniform stretch.

✗ Velocity is zero, so acceleration is zero.
✓ Top of a throw: still g, downward.

✗ Bounce: change in momentum = $m(10-8)$.
✓ Directions opposite: $m(10+8)$.

✗ Same speed means same velocity.
✓ A change of direction changes velocity.

✗ A heavy parked truck has much momentum.
✓ At rest, momentum 0.

✗ A rocket pushes against the air.
✓ It pushes on its own exhaust.