Final Exam Model Paper 3

MODEL PAPER

Full marks 75 · 3 hours

Name _____ Class _____ Roll no. ____ Date _____ Marks _____ / 75

All questions are compulsory. Time: 3 hours.

A practice paper by Science Playground, made to CDC's Specification Grid 2082, Optional Mathematics, Grades 9–10; not an official CDC or exam board paper. Mark it yourself online: the QR code below opens the paper with its answers.

A full practice run of the Grade 9 final exam in Optional Mathematics: 75 marks in 3 hours, 17 questions, every one compulsory, in three groups. Group A is one question of 11 multiple-choice items; groups B and C have questions of two to four parts, with the marks of each part in brackets. Show your working on paper as in the exam hall, then tap your choice in group A, open each other part's answer and give yourself the marks its scheme allows. You need graph paper for Questions 10 and 14.

Group A Objective questions — 11 marks

Question 1 — 11 × 1 = 11

Choose the correct option for each of the following questions.

A. What is the leading coefficient of the polynomial $p(x) = 5 - 2x + 3x^4$?

(i) 3 (ii) 5 (iii) −2 (iv) 4

B. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$, what is $A - B$?

(i) $\begin{pmatrix} -1 & 2 \\ 2 & 3 \end{pmatrix}$

(ii) $\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$

(iii) $\begin{pmatrix} 3 & 2 \\ 4 & 5 \end{pmatrix}$

(iv) $\begin{pmatrix} 1 & -2 \\ -2 & -3 \end{pmatrix}$

C. Which of these is equal to $1 + \cot^2\theta$?

(i) $\sec^2\theta$

(ii) $\operatorname{cosec}^2\theta$

(iii) $\tan^2\theta$

(iv) $\sin^2\theta$

D. What is the value of $\tan 225^\circ$?

(i) 1

(ii) −1

(iii) $\sqrt{3}$

(iv) $-\frac{1}{\sqrt{3}}$

E. Which of these angles is the largest?

(i) 1^c

(ii) 60°

(iii) 60^g

(iv) $\left(\frac{\pi}{4}\right)^c$

F. Under which transformation does **every** point of the line $y = x$ stay where it is?

(i) a reflection in the line $y = x$
(ii) a reflection in the line $y = -x$
(iii) a rotation through 180° about the origin
(iv) an enlargement with center 0 and scale factor 2

G. What is the equation of the line parallel to the X-axis through the point $(3, -4)$?

(i) $x = 3$ (ii) $y = -4$ (iii) $y = 3$ (iv) $x = -4$

H. What is the locus of a point that is always equally far from two fixed points A and B?

(i) a circle with AB as diameter
(ii) the perpendicular bisector of AB
(iii) the line AB
(iv) a circle with center A

I. If $\vec{p} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, what is $2\vec{p}$?

(i) $\begin{pmatrix} 4 \\ -6 \end{pmatrix}$
(ii) $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$
(iii) $\begin{pmatrix} 2 \\ -6 \end{pmatrix}$
(iv) $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$

J. The mean of a set of data is 20 and its standard deviation is 4. What is its coefficient of variation?

(i) 20% (ii) 5% (iii) 0.2% (iv) 80%

K. What is $\lim_{x \to 2}(x^2 + 1)$?

(i) 5
(ii) 4
(iii) 3
(iv) It does not exist.

Group B Within one content area — 40 marks

Question 2 — 4 marks

$A = \{1, 2, 3\}$ and $B = \{a, b, c\}$. Two relations from A to B are

$$R_1 = \{(1, a), (2, a), (3, b)\}, \qquad R_2 = \{(1, a), (1, b), (2, c)\}.$$

a. Define the Cartesian product $A \times B$. **[1]**

b. Which of R_1 and R_2 is a function from A to B? Give a reason for each. **[2]**

c. Show the interval $(-1, 3]$ on a number line. **[1]**

Question 3 — 3 marks

Nirmala writes the matrix

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

a. What type of square matrix is A? Give a reason. **[1]**

b. Nirmala says A is a scalar matrix. Is she right? Give a reason. **[1]**

c. Find $2A + I$, where I is the identity matrix of order 3. **[1]**

Question 4 — 3 marks

The graph of the polynomial $p(x) = x^2 - 4x + 1$ cuts the X-axis at $A(2+\sqrt{3}, 0)$ and $B(2-\sqrt{3}, 0)$.

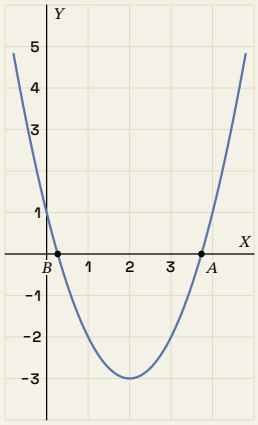


Figure 2 The graph of $y = x^2 - 4x + 1$, cutting the X-axis at A and B.

a. Write the roots of $p(x) = 0$ shown by the graph. **[1]**

b. Find the sum and the product of the two roots. **[2]**

Question 5 — 2 marks

A trigonometric expression $\cos(-60^\circ) + \sin 330^\circ$ is given.

a. Use $\cos(-\theta) = \cos\theta$ to find the value of $\cos(-60^\circ)$. **[1]**

b. Is there an angle between 90° and 180° whose sine is equal to $\sin 330^\circ$? Give a reason. **[1]**

Question 6 — 4 marks

A trigonometric expression $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$ is given.

a. Write $\sin^2\theta$ in terms of $\cos\theta$. **[1]**

b. Write $\sec\theta$ as the reciprocal of a trigonometric ratio. **[1]**

c. Prove that $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$. **[2]**

Question 7 — 3 marks

Arjun flies a kite (changa) on a string 50 m long, pulled straight. The kite is 40 m away from him along the ground, and the string makes an angle θ with the ground, so $\cos\theta = \frac{40}{50} = \frac{4}{5}$.

a. Find the values of $\tan\theta$ and $\operatorname{cosec}\theta$. **[2]**

b. Find the value of $\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}$. **[1]**

Question 8 — 3 marks

A straight path on a park's plan has the equation $3x - 4y + 20 = 0$.

a. Write the slope of a line $ax + by + c = 0$. **[1]**

b. Reduce the equation of the path to the double-intercept form, and write its x-intercept and y-intercept. **[2]**

Question 9 — 4 marks

$A(1, 0)$ and $B(-1, 0)$ are two fixed points.

a. A point P moves so that $PA^2 + PB^2 = 20$. Find the equation of its locus. **[2]**

b. Find the point that divides the line segment joining $C(1, -2)$ and $D(4, 7)$ externally in the ratio 2 : 1. **[2]**

Question 10 — 4 marks

Triangle ABC has vertices $A(2, 4)$, $B(4, 3)$ and $C(3, 6)$. It is rotated through 180° about the point $(1, 2)$.

a. Write the image of a point (x, y) under a rotation through 180° about a point (a, b). **[1]**

b. Find the coordinates of A', B' and C'. **[1]**

c. Draw $\triangle ABC$ and its image on the same graph. **[2]**

Question 11 — 3 marks

Tenzing walks across a field from A to B and then from B to C, where $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ (in units of 10 m).

a. State the triangle law of vector addition. **[1]**

b. Find $\overrightarrow{AC}$ and its magnitude. **[1]**

c. Tenzing says: "I walked $5 + \sqrt{29} \approx 10.39$ units, so $|\overrightarrow{AC}|$ must be 10.39 too." Is he right? Give a reason. **[1]**

Question 12 — 3 marks

The box-and-whisker plots show the marks of two sections of Grade 9 in a test.

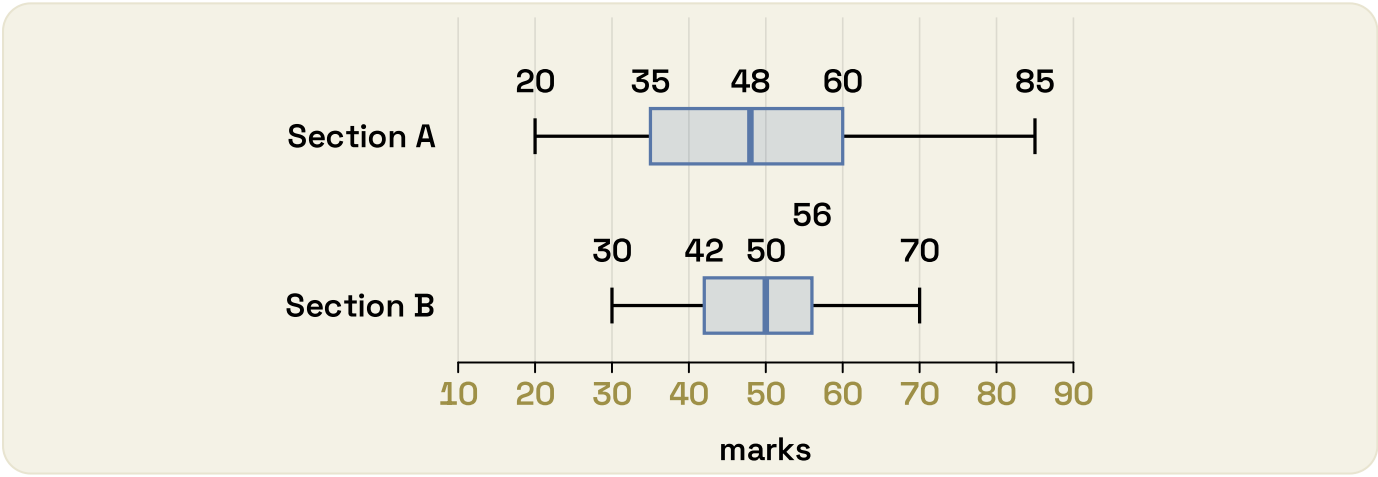


Figure 4 The marks of Sections A and B.

a. Write the five numbers a box-and-whisker plot is drawn from. **[1]**

b. Find the quartile deviation of the marks of Section A. **[1]**

c. The class teacher says the middle half of Section B's marks are much closer together than Section A's. Is she right? Give a reason from the plots. **[1]**

Question 13 — 4 marks

An infinite series $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \cdots$ is given.

a. What is meant by the sum of an infinite series? **[1]**

b. Anju says: "0.333... is always a little less than $\frac{1}{3}$, however many 3s it has, so it can never equal $\frac{1}{3}$." Is she right? Give a reason. **[1]**

c. Find the first four partial sums, and the number the series heads for. **[1]**

d. Does the series $1 + 2 + 3 + 4 + \cdots$ have a sum? Give a reason. **[1]**

Group C Across content areas — 24 marks

Question 14 — 6 marks

On a map of a village (1 unit = 1 km), a straight road passes through $A(-2,\,0)$ and $B(0,\,4)$. A point $P(k,\,2k+4)$ moves along it.

a. Find the equation of the road, and show that P lies on it for every value of k. **[2]**

b. The slope of OP, from the origin to P, is $\frac{2k+4}{k}$. What does it head for as $k \to \infty$? **[1]**

c. Find P when $k = -1$, and its distance from the origin. **[1]**

d. A farmhouse is at $F(1,\,3)$. Write the inequality for the side of the road that contains the origin, and find whether the farmhouse is on that side. Show it on a graph. **[2]**

Question 15 — 6 marks

Prakriti makes tile patterns: her xth pattern uses $f(x) = x^2 + 1$ tiles, for $x = 1,\,2,\,3,\,4,\,5,\,6,\,7$.

a. Write f as a set of ordered pairs, and write its range. **[2]**

b. Is f one-to-one on $\{1,\,2,\,\ldots,\,7\}$? Would it still be one-to-one if -1 were added to the domain? Give reasons. **[1]**

c. Find the quartile deviation of the numbers of tiles in the seven patterns, and its coefficient. **[3]**

Question 16 — 6 marks

The minute hand of a big wall clock at a school in Kathmandu is 42 cm long. In 20 minutes it turns through 120°. (Take $\pi = \frac{22}{7}$.)

a. Convert 120° into grades and into radians. **[2]**

b. How far does the tip of the minute hand travel in 20 minutes, and in one hour? **[2]**

c. The hand starts at 12. Shanti says that 20 minutes later its tip is below the level of the clock's center. Is she right? Use $\cos 120^\circ$ to decide. **[1]**

d. For shorter and shorter times, the hand turns through smaller and smaller angles θ (in radians). What does $\frac{\text{arc}}{\theta}$ head for as $\theta \to 0$? Give a reason. **[1]**

Question 17 — 6 marks

Rabina draws $\triangle ABC$ with $A(1,\,4)$, $B(3,\,1)$, $C(5,\,3)$ and its image $\triangle A'B'C'$ with $A'(4,\,1)$, $B'(1,\,3)$, $C'(3,\,5)$ under a single transformation.

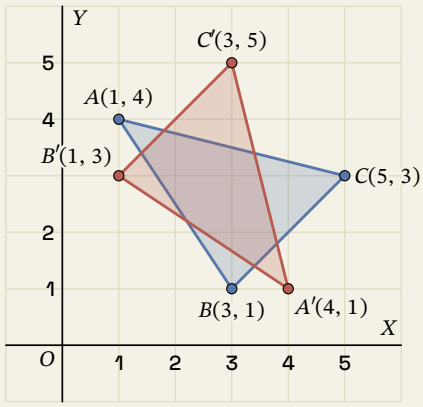


Figure 6 $\triangle ABC$ (blue) and its image $\triangle A'B'C'$ (red).

a. Rabina says that $\overrightarrow{AA'}$ and $\overrightarrow{BB'}$ are parallel. Is she right? Give a reason. **[1]**

b. Find the magnitudes of $\overrightarrow{AA'}$ and $\overrightarrow{BB'}$, and of $\overrightarrow{CC'}$. **[2]**

c. Find the single transformation that maps $\triangle ABC$ to $\triangle A'B'C'$. Write its rule and check it with every vertex. **[3]**