Final Exam Model Paper 2

ANSWER KEY

Full marks 75 · 3 hours

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

A practice paper by Science Playground, made to CDC's Specification Grid 2082, Optional Mathematics, Grades 9–10; not an official CDC or exam board paper.

Group A Objective questions — 11 marks

Question 1 — 11 × 1 = 11

A. (ii) $3 - \sqrt{2}$

B. (i) 11

C. (i) $\left(\frac{180}{\pi}\right)^\circ$

D. (i) $\frac{\sqrt{3}}{2}$

E. (iii) $\sin\theta + \cos\theta = 1$

F. (iii) a rotation through 180° about the origin

G. (i) 6

H. (i) $\frac{y_2 - y_1}{x_2 - x_1}$

I. (i) $(4, -2)$

J. (i) 15

K. (ii) 1

Group B Within one content area — 40 marks

Question 2 — 4 marks

a. A relation R in a set A is reflexive if **every element of A is related to itself**: $(x, x) \in R$ for every $x \in A$. **[1]**

Marking scheme

- $(x, x) \in R$ for every element x of the set.

b. **Reflexive:** $(1, 1)$, $(2, 2)$ and $(3, 3)$ are all in R, so it is reflexive ✓. **[2]**

Symmetric: $(1, 2) \in R$ but $(2, 1) \notin R$ (2 is not less than or equal to 1), so it is not symmetric.

She is only half right: R is reflexive but not symmetric.

Marking scheme

- Reflexive, since $(1, 1), (2, 2), (3, 3) \in R$.
- Not symmetric, since $(1, 2) \in R$ but $(2, 1) \notin R$; so she is only partly right.

c. Swap each pair: $R^{-1} = \{(1,1), (2,1), (3,1), (2,2), (3,2), (3,3)\}$. It is the relation "is greater than or equal to". **[1]**

Marking scheme

- R^{-1} written correctly ("is greater than or equal to").

Question 3 — 3 marks

a. **Yes.** It is square, and the elements on either side of the main diagonal are equal: $m_{12} = m_{21} = 4$. So M is its own mirror image in the main diagonal, whatever a is. **[1]**

Marking scheme

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- Symmetric, since $m_{12} = m_{21} = 4$.

b. Equal matrices have equal corresponding elements, so every element of $N - M$ is 0: $N - M$ **is the zero (null) matrix** $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. **[1]**

Marking scheme

- A zero matrix, since corresponding elements are equal.

c. Corresponding elements: $a + 1 = 5$, so $a = 4$; $b - 2 = 4$, so $b = 6$. **[1]**

Marking scheme

- $a = 4$ and $b = 6$.

Question 4 — 3 marks

a. $\sqrt{5} + \sqrt{3}$ **[1]**

Marking scheme

- $\sqrt{5} + \sqrt{3}$.

b. **[2]**

$$\begin{aligned} \frac{4}{\sqrt{5} - \sqrt{3}} &= \frac{4(\sqrt{5} + \sqrt{3})}{(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})} \\ &= \frac{4(\sqrt{5} + \sqrt{3})}{5 - 3} \\ &= 2\sqrt{5} + 2\sqrt{3} \end{aligned}$$

With $\sqrt{5} \approx 2.236$ and $\sqrt{3} \approx 1.732$: about $4.47 + 3.46 = 7.94$ m.

Marking scheme

- Top and bottom multiplied by $\sqrt{5} + \sqrt{3}$, the denominator $5 - 3 = 2$.
- $2\sqrt{5} + 2\sqrt{3}$ m (about 7.94 m).

Question 5 — 2 marks

a. **[1]**

$$\tan 135^\circ = \tan(180^\circ - 45^\circ) = -\tan 45^\circ = -1$$

Marking scheme

- $\tan 135^\circ = -\tan 45^\circ = -1$.

b. $\tan(-\theta) = -\tan\theta$, so $\tan(-45^\circ) = -\tan 45^\circ = -1 = \tan 135^\circ$. The angle is **-45°**. **[1]**

Marking scheme

- -45°, since $\tan(-45^\circ) = -1$.

Question 6 — 4 marks

a. $\cot\theta = \frac{\cos\theta}{\sin\theta}$ **[1]**

Marking scheme

- $\cot\theta = \frac{\cos\theta}{\sin\theta}$.

b. $\operatorname{cosec}^2\theta - \cot^2\theta = 1$ **[1]**

Marking scheme

- 1.

c. **[2]**

$$\begin{aligned} \text{LHS} &= \frac{\sec^2\theta}{\operatorname{cosec}^2\theta} = \frac{\frac{1}{\cos^2\theta}}{\frac{1}{\sin^2\theta}} \\ &= \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta = \text{RHS} \end{aligned}$$

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Proved.

Marking scheme

- $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ used.
- Simplified to $\frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$.

Question 7 — 3 marks

a. In a right-angled triangle with perpendicular 7 and base 24, the hypotenuse is $\sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25$. So **[2]**

$$\sin\theta = \frac{7}{25}, \qquad \cos\theta = \frac{24}{25}.$$

Marking scheme

- The hypotenuse 25 and $\sin\theta = \frac{7}{25}$.
- $\cos\theta = \frac{24}{25}$.

b. **[1]**

$$25\left(\frac{7}{25} + \frac{24}{25}\right) = 7 + 24 = 31$$

Marking scheme

- 31.

Question 8 — 3 marks

a. $\frac{x}{a} + \frac{y}{b} = 1$, where a is the x-intercept and b the y-intercept. **[1]**

Marking scheme

- $\frac{x}{a} + \frac{y}{b} = 1$.

b. The intercepts are equal, $b = a$: $\frac{x}{a} + \frac{y}{a} = 1$, that is, $x + y = a$. It passes through $(3,\ 4)$: $3 + 4 = a$, so $a = 7$. **[2]**

The canal is $x + y = 7$ ($x + y - 7 = 0$).

Marking scheme

- $x + y = a$ with $(3,\ 4)$ put in: $a = 7$.
- $x + y = 7$.

Question 9 — 4 marks

a. Let $P(x,\ y)$. Its distance from the X-axis is $|y|$, so $PA = |y|$: **[2]**

$$\sqrt{x^2 + (y-3)^2} = |y|$$
$$x^2 + y^2 - 6y + 9 = y^2$$
$$x^2 - 6y + 9 = 0$$

Marking scheme

- $\sqrt{x^2 + (y-3)^2} = |y|$ set up and squared.
- $x^2 - 6y + 9 = 0$.

b. Let it divide CD in the ratio m_1:m_2. On the X-axis $y = 0$: **[2]**

$$\frac{m_1 \times 6 + m_2 \times (-3)}{m_1 + m_2} = 0 \Rightarrow 6m_1 = 3m_2 \Rightarrow m_1 : m_2 = 1 : 2.$$

Then $x = \frac{1\times5+2\times2}{1+2} = \frac{9}{3} = 3$. **The ratio is 1 : 2, at the point $(3,\ 0)$.**

Marking scheme

- The ratio 1 : 2 from $y = 0$.

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- The point $(3,\ 0)$.

Question 10 — 4 marks

a. $P(x,\ y) \to P'(x,\ 2b - y)$ **[1]**

Marking scheme

- $(x,\ y) \to (x,\ 2b - y)$.

b. Here $b = 2$, so $(x,\ y) \to (x,\ 4 - y)$: $A'(1,\ 5), B'(4,\ 4), C'(2,\ 3)$. **[1]**

Marking scheme

- $A'(1,\ 5), B'(4,\ 4), C'(2,\ 3)$.

c. **[2]**

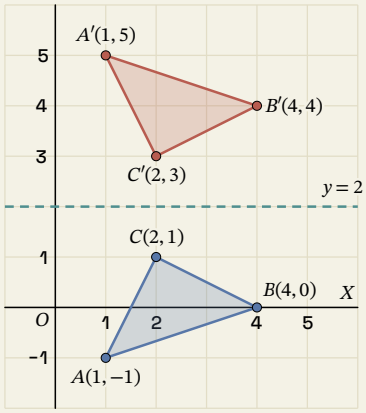


Figure 1 $\triangle ABC$ (blue) and its image $\triangle A'B'C'$ (red) in the line $y = 2$.

Each vertex and its image are the same distance from $y = 2$, on opposite sides.

Marking scheme

- $\triangle ABC$ plotted and labelled.
- $\triangle A'B'C'$ plotted and labelled, with the line $y = 2$.

Question 11 — 3 marks

a. It goes from $(-1,\ -2)$ to $(3,\ 1)$: $\vec{m} = \begin{pmatrix} 3 - (-1) \\ 1 - (-2) \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$. **[1]**

Marking scheme

- $\vec{m} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

b. $|\vec{m}| = \sqrt{4^2 + 3^2} = 5$, so $\hat{m} = \frac{1}{5}\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{4}{5} \\ \frac{3}{5} \end{pmatrix}$. **[1]**

Marking scheme

- $\hat{m} = \begin{pmatrix} \frac{4}{5} \\ \frac{3}{5} \end{pmatrix}$.

c. **No.** $-\vec{m} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}$ has the same magnitude, 5, but points the **opposite** way: it is the negative vector of $\vec{m}$, from $(3,\ 1)$ back to $(-1,\ -2)$. **[1]**

Marking scheme

- Same magnitude but opposite direction.

Question 12 — 3 marks

a. $Q_1 = \left(\frac{N+1}{4}\right)^{\text{th}}$ item, where $N = \Sigma f$. **[1]**

Marking scheme

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- $Q_1 = \left(\frac{N+1}{4}\right)^{\text{th}}$ item.

b. **[1]**

x	0	1	2	3	4
f	3	8	13	4	2
c.f.	3	11	24	28	30

$\frac{30+1}{4} = 7.75$. The c.f. just greater than 7.75 is 11, for $x = 1$: $Q_1 = 1$ **child**.

Marking scheme

- The 7.75th item, $Q_1 = 1$.

c. **No.** Q_3 is the $\frac{3(30+1)}{4} = 23.25$th item; the c.f. just greater than that is 24, for $x = 2$, so $Q_3 = 2$. **[1]**
Quartiles divide the households into four equal groups by count, so they depend on the frequencies, not on where the values sit between the smallest and the largest.

Marking scheme

- No: Q_3 is the 23.25th item, which is 2; quartiles depend on the frequencies.

Question 13 — 4 marks

a. x takes values closer and closer to 3 from the right, greater than 3 (3.1, 3.01, 3.001, ...), never equal to 3. **[1]**

Marking scheme

- x approaches 3 from values greater than 3.

b. **No.** They agree for every x except 3, but $f(3) = \frac{0}{0}$ has no value while $g(3) = 6$. A function is fixed by its domain as well as its rule, and 3 is not in the domain of f: its graph is the line $y = x + 3$ with a hole at $(3, 6)$. **[1]**

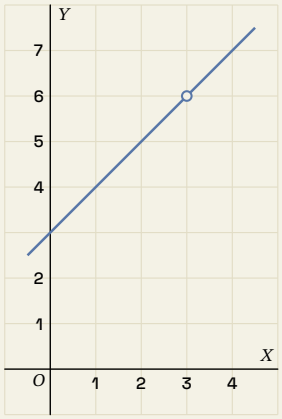


Figure 3 $y = \frac{x^2-9}{x-3}$: the line $y = x + 3$ with a hole at $(3, 6)$.

Marking scheme

- No: f has no value at $x = 3$ (0/0), while $g(3) = 6$.

c. **[1]**

x	2.9	2.99	2.999	3.001	3.01	3.1
$f(x)$	5.9	5.99	5.999	6.001	6.01	6.1

From both sides $f(x)$ heads for 6: $\lim_{x \to 3} f(x) = 6$.

Marking scheme

- The table filled and the limit 6.

d. Putting $x = 3$ gives $\frac{9-9}{3-3} = \frac{0}{0}$, an indeterminate form, so $f(3)$ has no value. The limit asks what $f(x)$ heads for **near** 3, which is why it is found from values on both sides. **[1]**

Marking scheme

- $x = 3$ gives $\frac{0}{0}$, which is indeterminate.

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Group C Across content areas — 24 marks

Question 14 — 6 marks

a. By the section formula with $m_1 = k, m_2 = 1$: **[2]**

$$x = \frac{k \times 4 + 1 \times (-2)}{k+1} = \frac{4k-2}{k+1}, \qquad y = \frac{k \times 0 + 1 \times 6}{k+1} = \frac{6}{k+1}.$$

Marking scheme

- $x = \frac{4k-2}{k+1}$.
- $y = \frac{6}{k+1}$.

b. $\frac{4k-2}{k+1} = \frac{4-\frac{2}{k}}{1+\frac{1}{k}} \to \frac{4}{1} = 4$ and $\frac{6}{k+1} \to 0$. So P heads for $(4,\,0)$**, the point B**: the larger the first part of the ratio, the closer P is to B. (For $k = 100$, P is about $(3.94,\,0.06)$.) **[1]**

Marking scheme

- P heads for $(4,\,0)$, that is, B.

c. In double-intercept form: the line through $B(4,\,0)$ with slope $\frac{0-6}{4-(-2)} = -1$ meets the Y-axis at $y = 4$, so $\frac{x}{4} + \frac{y}{4} = 1$, that is, $x + y = 4$. Check with A: $-2 + 6 = 4$ ✓. **[1]**

Marking scheme

- $x + y = 4$.

d. Multiply top and bottom by $\sqrt{2} - 1$, since $(\sqrt{2}+1)(\sqrt{2}-1) = 2 - 1 = 1$: **[2]**

$$x = \frac{4\sqrt{2}-2}{\sqrt{2}+1} = (4\sqrt{2}-2)(\sqrt{2}-1) = 8 - 4\sqrt{2} - 2\sqrt{2} + 2 = 10 - 6\sqrt{2}$$
$$y = \frac{6}{\sqrt{2}+1} = 6(\sqrt{2}-1) = 6\sqrt{2} - 6$$

Then $x + y = 10 - 6\sqrt{2} + 6\sqrt{2} - 6 = 4$ ✓, so P lies on AB. (P is about $(1.51,\,2.49)$.)

Marking scheme

- $x = 10 - 6\sqrt{2}$ and $y = 6\sqrt{2} - 6$, the denominators rationalized.
- $x + y = 4$, so P is on AB.

Question 15 — 6 marks

a. **[2]**

$$f = \{(-2,4), (-1,1), (0,0), (1,1), (2,4)\}, \qquad \text{range} = \{0, 1, 4\}.$$

Marking scheme

- f as the five ordered pairs.
- Range $\{0,\,1,\,4\}$.

b. **No, it is into.** Its range $\{0,\,1,\,4\}$ is not the whole co-domain B: 2 and 3 have no pre-image (no whole number in A squares to 2 or 3). **[1]**

Marking scheme

- Into, not onto, since 2 and 3 in B have no pre-image.

c. $\overline{X} = \frac{4+1+0+1+4}{5} = \frac{10}{5} = 2$. **[3]**

X	4	1	0	1	4
$d = X - \overline{X}$	2	-1	-2	-1	2
d^2	4	1	4	1	4

$$\sigma = \sqrt{\frac{\Sigma d^2}{N}} = \sqrt{\frac{14}{5}} = \sqrt{2.8} \approx 1.67$$

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Marking scheme

- The mean, 2.
- $\Sigma d^2 = 14$.
- $\sigma = \sqrt{2.8} \approx 1.67$.

Question 16 — 6 marks

a. **[2]**

$$150^{\text{g}} = \left(150 \times \frac{9}{10}\right)^{\circ} = 135^{\circ}, \qquad 150^{\text{g}} = \left(150 \times \frac{\pi}{200}\right)^{\text{c}} = \left(\frac{3\pi}{4}\right)^{\text{c}}$$

Marking scheme

- 135∘.
- $\frac{3\pi}{4}$ radians.

b. **[2]**

$$l = r\theta = 21 \times \frac{3}{4} \times \frac{22}{7} = 49.5 \text{ cm}$$

For 66 cm: $\theta = \frac{l}{r} = \frac{66}{21} = \frac{22}{7}$ radians, which is π radians, **180∘**: half a turn.

Marking scheme

- 49.5 cm in one second.
- $\theta = \frac{66}{21} = \pi$ radians = 180∘.

c. **No.** $150^{\text{g}} = 135^{\circ}$, so $\sin 150^{\text{g}} = \sin 135^{\circ} = \sin(180^{\circ} - 45^{\circ}) = \frac{1}{\sqrt{2}} \approx 0.71$, but $\sin 150^{\circ} = \sin(180^{\circ} - 30^{\circ}) = \frac{1}{2}$. They are different angles, measured in different units. **[1]**

Marking scheme

- No: $\sin 150^{\text{g}} = \sin 135^{\circ} = \frac{1}{\sqrt{2}}$, while $\sin 150^{\circ} = \frac{1}{2}$.

d. **No.** The distance is $49.5t$ cm, which grows past any number we name as t grows without bound: it tends to infinity, and infinity is not a number, so there is no limit. **[1]**

Marking scheme

- No: $49.5t \to \infty$, so it heads for no number.

Question 17 — 6 marks

a. $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\overrightarrow{A'B'} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$: both have magnitude 2, but one points right and the other up. **He is right**: the vectors are not equal, though their magnitudes are. **[1]**

Marking scheme

- $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \overrightarrow{A'B'} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$: same magnitude, different direction, so he is right.

b. **[2]**

$$\overrightarrow{BC} = \begin{pmatrix} 3-3 \\ 5-2 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \qquad \overrightarrow{B'C'} = \begin{pmatrix} -3-0 \\ 3-3 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$$

$|\overrightarrow{BC}| = 3$ and $|\overrightarrow{B'C'}| = 3$.

Marking scheme

- $\overrightarrow{BC} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ and $\overrightarrow{B'C'} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$.
- Both magnitudes 3.

c. **Not about the origin:** +90∘ about O would send $A(1, 2)$ to $(-2, 1)$, not $(0, 1)$. **[3]**

The center is as far from each point as from its image, so it lies on the locus of points equidistant from A and A', and from B and B'. For $P(x, y)$:

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$$PA = PA' : (x-1)^2 + (y-2)^2 = x^2 + (y-1)^2 \Rightarrow x + y = 2$$
$$PB = PB' : (x-3)^2 + (y-2)^2 = x^2 + (y-3)^2 \Rightarrow 3x - y = 2$$

Adding: $4x = 4, x = 1, y = 1$. **The center is** $(1, 1)$**.**

The angle: from the center, A is 1 unit up, $(0, 1)$, and A' is 1 unit left, $(-1, 0)$: a quarter turn anticlockwise. So the rotation is **+90∘ (anticlockwise) about** $(1, 1)$. Check with the rule $(x, y) \to (-(y-1)+1, (x-1)+1) = (2-y, x)$: $C(3, 5) \to (-3, 3) = C'$ ✓.

Marking scheme

- The center found from equal distances (or by the rule about a point): $(1, 1)$.
- The angle 90∘.
- The direction anticlockwise (+90∘), checked with a vertex.

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