Final Exam Model Paper 1

MODEL PAPER

Full marks 75 · 3 hours

Name _____ Class _____ Roll no. ____ Date _____ Marks _____ / 75

All questions are compulsory. Time: 3 hours.

A practice paper by Science Playground, made to CDC's Specification Grid 2082, Optional Mathematics, Grades 9–10; not an official CDC or exam board paper. Mark it yourself online: the QR code below opens the paper with its answers.

A full practice run of the Grade 9 final exam in Optional Mathematics: 75 marks in 3 hours, 17 questions, every one compulsory, in three groups. Group A is one question of 11 multiple-choice items; groups B and C have questions of two to four parts, with the marks of each part in brackets. Show your working on paper as in the exam hall, then tap your choice in group A, open each other part's answer and give yourself the marks its scheme allows. You need graph paper for Questions 10 and 14.

Group A Objective questions — 11 marks

Question 1 — 11 × 1 = 11

Choose the correct option for each of the following questions.

A. Which interval is the set $\{x: -2 \le x < 3, x \in R\}$?

(i) $[-2, 3]$ (ii) $(-2, 3)$ (iii) $[-2, 3)$ (iv) $(-2, 3]$

B. If $f(x) = x^2 - 3x + 2$, what is $f(-2)$?

(i) 12 (ii) 0 (iii) 4 (iv) −8

C. In the centesimal system, how many grades make one right angle?

(i) 90^g (ii) 100^g (iii) 180^g (iv) 200^g

D. In a right-angled triangle, $\sec\theta = \frac{13}{12}$. What is the value of $\tan\theta$?

(i) $\frac{5}{12}$ (ii) $\frac{12}{5}$
(iii) $\frac{5}{13}$ (iv) $\frac{13}{5}$

E. If $\cos(180^\circ - A) = \frac{1}{2}$ and $0^\circ < A < 180^\circ$, what is the value of A?

(i) 30° (ii) 60° (iii) 120° (iv) 150°

F. Under which transformation does the point $(3, -2)$ stay where it is?

(i) a reflection in the X-axis
(ii) a reflection in the line $x = 3$
(iii) a rotation through +90° about the origin
(iv) an enlargement with center O and scale factor 2

G. What is the slope of the line through $(2, -1)$ and $(5, 5)$?

(i) 2 (ii) $\frac{1}{2}$
(iii) −2 (iv) $\frac{4}{3}$

H. A point P divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m_1{:}m_2$. What is its x-coordinate?

(i) $\frac{m_1x_2 + m_2x_1}{m_1 + m_2}$
(ii) $\frac{m_1x_1 + m_2x_2}{m_1 + m_2}$
(iii) $\frac{m_1x_2 - m_2x_1}{m_1 - m_2}$
(iv) $\frac{x_1 + x_2}{2}$

I. Which of these is a unit vector?

(i) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(ii) $\begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$

(iii) $\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$

(iv) $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

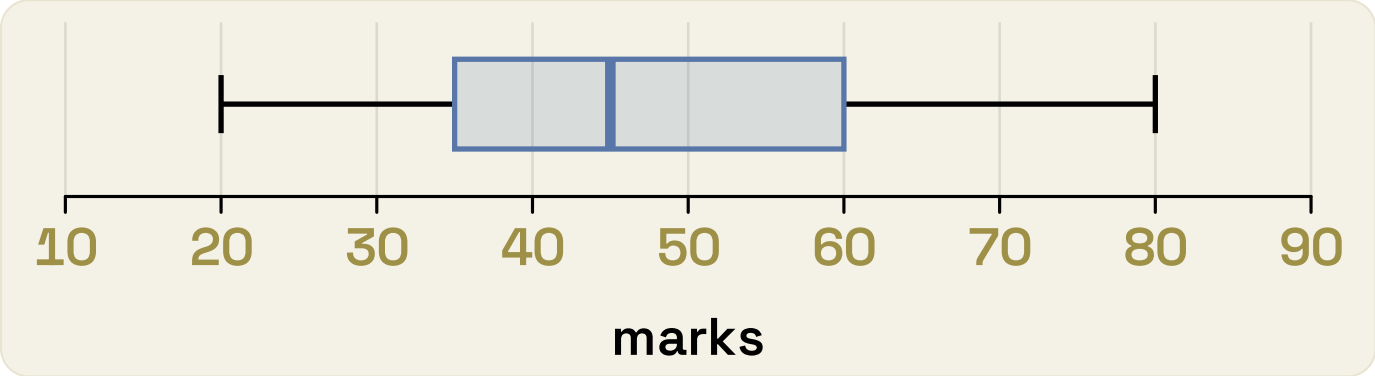


Figure 1 The marks of Grade 9 students in a class test.

J. The box-and-whisker plot above shows the marks of the Grade 9 students of a school in a class test. What is the median mark?

(i) 35 (ii) 45 (iii) 60 (iv) 50

K. Which of these has no value (is undefined)?

(i) $\frac{0}{5}$ (ii) $\frac{5}{0}$

(iii) 5×0 (iv) $5 - 5$

Group B Within one content area — 40 marks

Question 2 — 4 marks

Tara's number machine takes a number from $A = \{-2, -1, 1, 2\}$, squares it and adds 1, giving a number in $B = \{1, 2, 3, 4, 5\}$. So it is the function $f: A \to B$, $f(x) = x^2 + 1$.

a. Define a function from a set A to a set B. **[1]**

b. Tara says the inverse of f, which runs her machine backwards, is also a function. Write f and its inverse as sets of ordered pairs, and say whether she is right. **[2]**

c. For the next game the machine uses the rule $g(x) = 2x - 3$ on the real numbers. Show on a number line the values of x for which $g(x) > 1$. **[1]**

Question 3 — 3 marks

Two tea stalls in Ilam record the cups of tea and coffee they sold on Saturday (first row) and Sunday (second row). The first column is tea and the second coffee.

$$A = \begin{pmatrix} 12 & 8 \\ 15 & 10 \end{pmatrix} \text{ (first stall)}, \qquad B = \begin{pmatrix} 9 & 11 \\ 14 & 6 \end{pmatrix} \text{ (second stall)}$$

a. What does the element a_{21} of A tell? **[1]**

b. Is A a symmetric matrix? Give a reason. **[1]**

c. Find $A + B$ and say what it shows. **[1]**

Question 4 — 3 marks

Shristi's teacher writes the polynomial $p(x) = x^3 - 2x^2 - 5x + 6$ on the board.

a. Write the degree of $p(x)$. **[1]**

b. Divide $p(x)$ by $(x - 3)$ by synthetic division, and write the quotient and the remainder. What does the remainder tell Shristi about $x = 3$? **[2]**

Question 5 — 2 marks

A trigonometric expression $\cos 150^\circ + \sin 300^\circ$ is given.

a. Use $\cos(180^\circ - \theta) = -\cos\theta$ to write $\cos 150^\circ$ as a trigonometric ratio of an acute angle, and find its value. **[1]**

b. Find the angle between 180° and 270° whose sine is equal to $\sin 300^\circ$. **[1]**

Question 6 — 4 marks

A trigonometric expression $(\sec\theta - \cos\theta)(\operatorname{cosec}\theta - \sin\theta)$ is given.

a. Write the Pythagoras relation between $\sec\theta$ and $\tan\theta$. **[1]**

b. Write $\operatorname{cosec}\theta$ in terms of $\sin\theta$. **[1]**

c. Prove that $(\sec\theta - \cos\theta)(\operatorname{cosec}\theta - \sin\theta) = \sin\theta\cos\theta$. **[2]**

Question 7 — 3 marks

A ladder 17 m long leans against a wall at a building site in Butwal, and its top reaches 8 m up the wall. It makes an angle θ with the ground, so $\sin\theta = \frac{8}{17}$.

a. Find the values of $\cos\theta$ and $\cot\theta$. **[2]**

b. Find the value of $17\sin\theta + 8\cot\theta$. **[1]**

Question 8 — 3 marks

On a map of a town, with the school at the origin and 1 unit = 1 km, a straight road is drawn. The perpendicular from the school to the road is 4 km long and makes an angle of 30° with the X-axis.

a. Write the normal (perpendicular) form of the equation of a straight line. **[1]**

b. Find the equation of the road. **[2]**

Question 9 — 4 marks

On the same map, a mobile tower stands at $T(2, -1)$ and its signal reaches 5 km.

a. Find the equation of the locus of the points exactly 5 km from the tower, where the signal ends. **[2]**

b. A road runs straight from $A(-3, 4)$ to $B(7, -1)$, and a shop P on it divides AB internally in the ratio 3 : 2. Find the coordinates of P. **[2]**

Question 10 — 4 marks

Triangle PQR has vertices $P(1, 1)$, $Q(4, 2)$ and $R(2, 4)$. It is reflected in the line $y = -x$.

a. Write the image of a point (x, y) under a reflection in the line $y = -x$. **[1]**

b. Find the coordinates of the image points P', Q' and R'. **[1]**

c. Draw $\triangle PQR$ and its image $\triangle P'Q'R'$ on the same graph. **[2]**

Question 11 — 3 marks

A drone flies in a straight line from $A(2, -1)$ to $B(5, 3)$ over a field marked in units of 10 m.

a. Write $\overrightarrow{AB}$ in column vector form. **[1]**

b. Find the magnitude of $\overrightarrow{AB}$. How far did the drone fly? **[1]**

c. A second drone flies from $C(0,\,0)$ to $D(6,\,8)$. Anisha says $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are equal vectors, because they point the same way. Is she right? Give a reason. **[1]**

Question 12 — 3 marks

The waiting times, in minutes, of 11 patients at a health post in Dhading one morning were:

12	7	15	9	20	11	8	25	14	10	18

a. Write the formula for the position of the upper quartile (Q_3) of an individual series. **[1]**

b. Find the upper quartile of the waiting times. **[1]**

c. Mingma says: "If the longest wait had been 60 minutes instead of 25, the quartile deviation would not change, but the standard deviation would." Is she right? Give a reason. **[1]**

Question 13 — 4 marks

The graph of a function $y = f(x)$ is given.

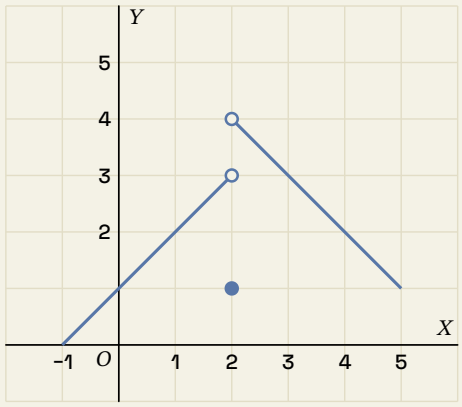


Figure 4 The graph of $y = f(x)$: open circles at $(2,\,3)$ and $(2,\,4)$, and a filled dot at $(2,\,1)$.

a. What is meant by the left-hand limit of a function at $x = a$? **[1]**

b. Rojina says: "$\frac{0}{0} = 1$, because any number divided by itself is 1." Is she right? Explain. **[1]**

c. Find $\lim_{x \to 2^-} f(x)$. **[1]**

d. Does $\lim_{x \to 2} f(x)$ exist? Give a reason. **[1]**

Group C Across content areas — 24 marks

Question 14 — 6 marks

A straight line l with slope m passes through the point $(2,\,5)$. Its equation is $y = mx + c$.

a. Show that the y-intercept of l is $c = 5 - 2m$ and its x-intercept is $\frac{2m-5}{m}$. **[2]**

b. As the line gets steeper and steeper ($m \to \infty$), what does its x-intercept head for? **[1]**

c. Find the equation of l when $m = 3$. **[1]**

d. For $m = 3$, write the inequality for the side of l that contains the origin, and show that region on a graph. **[2]**

Question 15 — 6 marks

At a maths fair in Dhangadhi, a number machine follows the rule $f(x) = 5x - 3$ for the inputs $x = 1,\,2,\,3,\,4,\,5$.

a. Write f as a set of ordered pairs, and write its range. **[2]**

b. Is f a one-to-one function? Would its inverse be a function? Give a reason. **[1]**

c. Find the mean deviation from the mean of the five outputs, and its coefficient. **[3]**

Question 16 — 6 marks

The pendulum of a wall clock in Bharatpur is 50 cm long, and in one swing it turns through an angle of 30°. (Take $\pi = \frac{22}{7}$.)

a. Convert 30° into grades and into radians. **[2]**

b. Find the length of the arc the bob travels in one swing. How far does it travel in a minute if it makes 40 swings a minute? **[2]**

c. Dipesh says: "If the pendulum swung through 60° instead, both the arc and the sine of the angle would be doubled." Is he right? Give a reason. **[1]**

d. Suppose the bob always travels the same arc, about 26.19 cm, but the pendulum is made longer and longer ($r \to \infty$). What does the angle of the swing head for? **[1]**

Question 17 — 6 marks

A sign painter in Janakpur enlarges a triangle of a shop's logo. $\triangle PQR$ with $P(2,\,1)$, $Q(4,\,1)$, $R(4,\,3)$ becomes $\triangle P'Q'R'$ with $P'(3,\,1)$, $Q'(7,\,1)$, $R'(7,\,5)$.

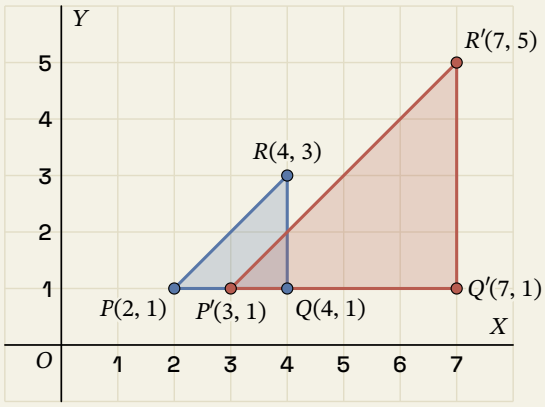


Figure 6 $\triangle PQR$ **(blue) and its enlarged image** $\triangle P'Q'R'$ **(red).**

a. Kamal says that $\overrightarrow{PP'}$ and $\overrightarrow{QQ'}$ are equal vectors, because each goes from a point to its image. Is he right? Give a reason. **[1]**

b. Find $\overrightarrow{PR}$ and $\overrightarrow{P'R'}$ in column form, and their magnitudes. How are they related? **[2]**

c. Find the scale factor and the center of the enlargement, and check your center with all three vertices. **[3]**