Final Exam Model Paper 1

ANSWER KEY

Full marks 75 · 3 hours

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

A practice paper by Science Playground, made to CDC's Specification Grid 2082, Optional Mathematics, Grades 9–10; not an official CDC or exam board paper.

Group A Objective questions — 11 marks

Question 1 — 11 × 1 = 11

A. **(iii)** $[-2, 3)$

B. **(i)** 12

C. **(ii)** 100^{g}

D. **(i)** $\frac{5}{12}$

E. **(iii)** 120°

F. **(ii)** a reflection in the line $x = 3$

G. **(i)** 2

H. **(i)** $\frac{m_1x_2+m_2x_1}{m_1+m_2}$

I. **(ii)** $\begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$

J. **(ii)** 45

K. **(ii)** $\frac{5}{0}$

Group B Within one content area — 40 marks

Question 2 — 4 marks

a. A function from A to B is a relation in which **every element of A is related to exactly one element of B.** **[1]**

Marking scheme

- Every element of A has one and only one image in B.

b. $f(-2) = 5, f(-1) = 2, f(1) = 2, f(2) = 5$, so **[2]**

$$f = \{(-2, 5), (-1, 2), (1, 2), (2, 5)\}, \quad f^{-1} = \{(5, -2), (2, -1), (2, 1), (5, 2)\}.$$

She is not right. In f^{-1} the element 2 is related to both −1 and 1 (and 5 to both −2 and 2), so it does not give one image to each element: f^{-1} is a relation but not a function. Run backwards, the machine can't tell which number went in.

Marking scheme

- f and f^{-1} written as ordered pairs.
- Not a function, because 2 (or 5) has two images in f^{-1}.

c. $2x - 3 > 1 \Rightarrow 2x > 4 \Rightarrow x > 2$: an open dot at 2 (2 itself is not included) and the line going right. **[1]**

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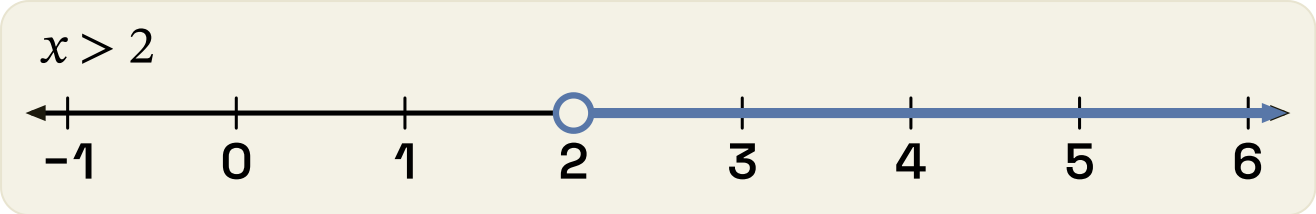


Figure 2 $x > 2$ on a number line.

Marking scheme

- $x > 2$ shown with an open dot at 2 and the ray to the right.

Question 3 — 3 marks

a. a_{21} is in the 2nd row and 1st column: $a_{21} = 15$. **The first stall sold 15 cups of tea on Sunday.** **[1]**

Marking scheme

- $a_{21} = 15$, the cups of tea the first stall sold on Sunday.

b. **No.** A symmetric matrix has $a_{ij} = a_{ji}$, but here $a_{12} = 8$ and $a_{21} = 15$. **[1]**

Marking scheme

- Not symmetric, since $a_{12} = 8 \neq a_{21} = 15$.

c. **[1]**

$$A + B = \begin{pmatrix} 12+9 & 8+11 \\ 15+14 & 10+6 \end{pmatrix} = \begin{pmatrix} 21 & 19 \\ 29 & 16 \end{pmatrix}$$

It shows the **two stalls' sales together**: 21 teas and 19 coffees on Saturday, 29 teas and 16 coffees on Sunday.

Marking scheme

- $A + B = \begin{pmatrix} 21 & 19 \\ 29 & 16 \end{pmatrix}$.

Question 4 — 3 marks

a. The highest power of x is 3, so the degree is **3** (a cubic polynomial). **[1]**

Marking scheme

- Degree 3.

b. Divide by $x - 3$, so use 3 at the left; the coefficients are 1, −2, −5, 6. **[2]**

3	1	−2	−5	6
		3	3	−6
	1	1	−2	0

Quotient $x^2 + x - 2$, remainder 0. Since the remainder is 0, $p(3) = 0$: **3 is a root** of $p(x) = 0$. Check: $27 - 18 - 15 + 6 = 0$ ✓.

Marking scheme

- The synthetic division, with quotient $x^2 + x - 2$.
- Remainder 0, so 3 is a root of $p(x) = 0$.

Question 5 — 2 marks

a. **[1]**

$$\cos 150^\circ = \cos(180^\circ - 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

Marking scheme

- $\cos 150^\circ = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$.

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b. $\sin 300^\circ = \sin(360^\circ - 60^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$. Between 180° and 270° (the third quadrant) the sine is negative, and $\sin(180^\circ + 60^\circ) = -\sin 60^\circ$. So the angle is **240°**. **[1]**

Marking scheme

- 240°, since $\sin 240^\circ = -\sin 60^\circ = \sin 300^\circ$.

Question 6 — 4 marks

a. $\sec^2\theta - \tan^2\theta = 1$ (that is, $1 + \tan^2\theta = \sec^2\theta$). **[1]**

Marking scheme

- $\sec^2\theta - \tan^2\theta = 1$.

b. $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ **[1]**

Marking scheme

- $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$.

c. **[2]**

$$\begin{aligned}\text{LHS} &= \left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{1}{\sin\theta} - \sin\theta\right)\\ &= \frac{1-\cos^2\theta}{\cos\theta} \times \frac{1-\sin^2\theta}{\sin\theta}\\ &= \frac{\sin^2\theta}{\cos\theta} \times \frac{\cos^2\theta}{\sin\theta}\\ &= \sin\theta\cos\theta = \text{RHS}\end{aligned}$$

Proved.

Marking scheme

- $\sec\theta$ and $\operatorname{cosec}\theta$ written as reciprocals and each bracket simplified to $\frac{\sin^2\theta}{\cos\theta}$ and $\frac{\cos^2\theta}{\sin\theta}$.
- Multiplied to $\sin\theta\cos\theta$.

Question 7 — 3 marks

a. The foot of the ladder is $\sqrt{17^2 - 8^2} = \sqrt{289 - 64} = \sqrt{225} = 15$ m from the wall. So **[2]**

$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{15}{17}, \qquad \cot\theta = \frac{\text{base}}{\text{perpendicular}} = \frac{15}{8}.$$

Marking scheme

- The base, 15 m, and $\cos\theta = \frac{15}{17}$.
- $\cot\theta = \frac{15}{8}$.

b. **[1]**

$$17 \times \frac{8}{17} + 8 \times \frac{15}{8} = 8 + 15 = 23$$

Marking scheme

- 23.

Question 8 — 3 marks

a. $x\cos\alpha + y\sin\alpha = p$, where p is the length of the perpendicular from the origin to the line and α the angle it makes with the X-axis. **[1]**

Marking scheme

- $x\cos\alpha + y\sin\alpha = p$.

b. Here $p = 4$ and $\alpha = 30^\circ$: **[2]**

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$$x\cos 30^\circ + y\sin 30^\circ = 4$$
$$\frac{\sqrt{3}}{2}x + \frac{1}{2}y = 4$$
$$\sqrt{3}x + y = 8$$

The road is $\sqrt{3}x + y - 8 = 0$.

Marking scheme

- $p = 4, \alpha = 30^\circ$ put in: $\frac{\sqrt{3}}{2}x + \frac{1}{2}y = 4$.
- $\sqrt{3}x + y = 8$.

Question 9

4 marks

a. Let $P(x, y)$ be such a point. Then $PT = 5$: **[2]**

$$\sqrt{(x-2)^2 + (y+1)^2} = 5$$
$$(x-2)^2 + (y+1)^2 = 25$$
$$x^2 - 4x + 4 + y^2 + 2y + 1 = 25$$
$$x^2 + y^2 - 4x + 2y - 20 = 0$$

The locus is a circle with center T and radius 5 km.

Marking scheme

- $\sqrt{(x-2)^2 + (y+1)^2} = 5$ from the distance formula.
- $x^2 + y^2 - 4x + 2y - 20 = 0$.

b. $m_1 = 3, m_2 = 2, (x_1, y_1) = (-3, 4), (x_2, y_2) = (7, -1)$: **[2]**

$$x = \frac{3 \times 7 + 2 \times (-3)}{3 + 2} = \frac{21 - 6}{5} = 3$$
$$y = \frac{3 \times (-1) + 2 \times 4}{3 + 2} = \frac{-3 + 8}{5} = 1$$

P(3, 1). It lies between A and B, as it should.

Marking scheme

- The section formula with the values put in correctly.
- P(3, 1).

Question 10

4 marks

a. $P(x, y) \to P'(-y, -x)$ **[1]**

Marking scheme

- $(x, y) \to (-y, -x)$.

b. $P(1, 1) \to P'(-1, -1), Q(4, 2) \to Q'(-2, -4), R(2, 4) \to R'(-4, -2)$. **[1]**

Marking scheme

- $P'(-1, -1), Q'(-2, -4), R'(-4, -2)$.

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c. [2]

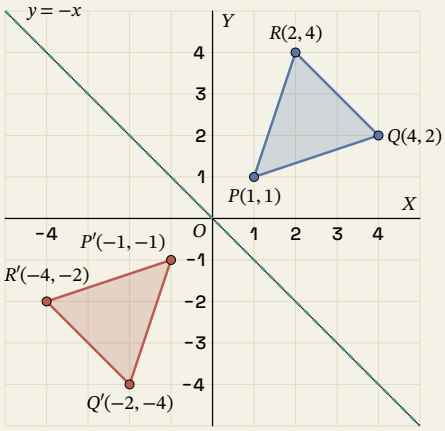


Figure 3 $\triangle PQR$ (blue) and its image $\triangle P'Q'R'$ (red) in the line $y = -x$.

Each image point is as far from the mirror line as its object, on the other side.

Marking scheme

- $\triangle PQR$ plotted and labelled.
- $\triangle P'Q'R'$ plotted and labelled, with the line $y = -x$.

Question 11 — 3 marks

a. [1]

$$\overrightarrow{AB} = \begin{pmatrix} 5-2 \\ 3-(-1) \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

Marking scheme

- $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

b. $|\overrightarrow{AB}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$ units, so the drone flew **50 m**. [1]

Marking scheme

- $|\overrightarrow{AB}| = 5$ units (50 m).

c. $\overrightarrow{CD} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} = 2\overrightarrow{AB}$: the same direction, but $|\overrightarrow{CD}| = 10$ while $|\overrightarrow{AB}| = 5$. **She is not right:** equal vectors need the same magnitude as well as the same direction. These are like vectors, not equal ones. [1]

Marking scheme

- Not equal: same direction but magnitudes 5 and 10 (like vectors).

Question 12 — 3 marks

a. $Q_3 = \left(\frac{3(N+1)}{4}\right)^{\text{th}}$ item, with the data in ascending order. [1]

Marking scheme

- $Q_3 = \left(\frac{3(N+1)}{4}\right)^{\text{th}}$ item.

b. In order: 7, 8, 9, 10, 11, 12, 14, 15, 18, 20, 25. $\frac{3(11+1)}{4} = 9$, so Q_3 is the 9th item: $Q_3 = 18$ **minutes**. [1]

Marking scheme

- $Q_3 = 18$ minutes (the 9th item in order).

c. **Yes.** The quartiles are the 3rd and 9th items in order (9 and 18 minutes); changing the largest value, the 11th, doesn't move them, so $\text{Q.D.} = \frac{18-9}{2} = 4.5$ minutes either way. The standard deviation uses every value's distance from the mean, so a 60 makes it much larger (from about 5.3 to about 14.2 minutes). [1]

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Marking scheme

- Yes: Q_1 and Q_3 don't depend on the largest value, but the standard deviation uses every value.

Question 13 — 4 marks

a. The left-hand limit is the number $f(x)$ gets as close as we like to as x takes values closer and closer to a from the left, less than a ($x \to a^-$), never equal to a. It is written $\lim_{x \to a^-} f(x)$. **[1]**

Marking scheme

- The value $f(x)$ heads for as $x \to a$ from values less than a.

b. **No.** If $\frac{0}{0} = m$, then $0 = 0 \times m$, which is true for every number m, not just 1. So $\frac{0}{0}$ can't be given one value: it is an indeterminate form. The rule "a number divided by itself is 1" holds only for numbers other than 0. **[1]**

Marking scheme

- No: $0 = 0 \times m$ for every m, so $\frac{0}{0}$ is indeterminate.

c. From the left of 2 the graph is the line $y = x + 1$, heading for the open circle at height 3: $\lim_{x \to 2^-} f(x) = 3$. **[1]**

Marking scheme

- 3.

d. From the right, the line $y = 6 - x$ heads for 4: $\lim_{x \to 2^+} f(x) = 4$. Since $3 \neq 4$, **the limit does not exist.** (The filled dot, $f(2) = 1$, doesn't matter for the limit.) **[1]**

Marking scheme

- No, because the left-hand limit 3 and the right-hand limit 4 are not equal.

Group C Across content areas — 24 marks

Question 14 — 6 marks

a. $(2, 5)$ is on the line: $5 = 2m + c$, so $c = 5 - 2m$.
The x-intercept is where $y = 0$: $0 = mx + 5 - 2m$, so $mx = 2m - 5$ and $x = \frac{2m-5}{m}$. **[2]**

Marking scheme

- $c = 5 - 2m$ from putting $(2, 5)$ in.
- x-intercept $\frac{2m-5}{m}$ from $y = 0$.

b. $\frac{2m-5}{m} = 2 - \frac{5}{m}$. As $m \to \infty$, $\frac{5}{m} \to 0$, so the x-intercept heads for **2**: the line becomes the vertical line $x = 2$. (For $m = 10, 100, 1000$ it is 1.5, 1.95, 1.995.) **[1]**

Marking scheme

- The x-intercept heads for 2.

c. $c = 5 - 2 \times 3 = -1$, so $y = 3x - 1$, that is, $3x - y - 1 = 0$. **[1]**

Marking scheme

- $y = 3x - 1$ (or $3x - y - 1 = 0$).

d. Put $(0, 0)$ in $3x - y - 1$: $0 - 0 - 1 = -1 < 0$. So the side containing the origin is $3x - y - 1 < 0$, that is, $y > 3x - 1$. **[2]**

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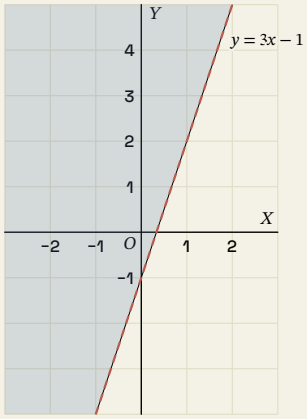


Figure 5 The region $y > 3x - 1$: the line $y = 3x - 1$ dashed (not included), the side holding the origin shaded.

Marking scheme

- $3x - y - 1 < 0$ (or $y > 3x - 1$) from testing the origin.
- The line drawn and the region holding the origin shaded.

Question 15

6 marks

a. $f(1) = 2, f(2) = 7, f(3) = 12, f(4) = 17, f(5) = 22$: **[2]**

$$f = \{(1, 2), (2, 7), (3, 12), (4, 17), (5, 22)\}, \qquad \text{range} = \{2, 7, 12, 17, 22\}.$$

Marking scheme

- f as the five ordered pairs.
- Range $\{2, 7, 12, 17, 22\}$.

b. **Yes, it is one-to-one**: different inputs give different outputs (each output is 5 more than the one before). So every output has exactly one input, and the inverse $\{(2, 1), (7, 2), (12, 3), (17, 4), (22, 5)\}$ gives each element one image: **it is a function too**. **[1]**

Marking scheme

- One-to-one, so the inverse relates each output to one input and is a function.

c. $\overline{X} = \frac{2+7+12+17+22}{5} = \frac{60}{5} = 12$. **[3]**

X	2	7	12	17	22
$\lvert X - \overline{X}\rvert$	10	5	0	5	10

$$\text{M.D.} = \frac{\Sigma|X - \overline{X}|}{N} = \frac{30}{5} = 6$$
$$\text{Coefficient of M.D.} = \frac{\text{M.D.}}{\overline{X}} = \frac{6}{12} = 0.5$$

Marking scheme

- The mean, 12.
- $\Sigma|X - \overline{X}| = 30$ and M.D. = 6.
- Coefficient of M.D. = 0.5.

Question 16

6 marks

a. **[2]**

$$30^\circ = \left(30 \times \frac{10}{9}\right)^{\text{g}} = \left(\frac{100}{3}\right)^{\text{g}} = 33\tfrac{1}{3}^{\text{g}}, \qquad 30^\circ = \left(30 \times \frac{\pi}{180}\right)^{\text{c}} = \left(\frac{\pi}{6}\right)^{\text{c}}$$

Marking scheme

- $33\frac{1}{3}$ grades.

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- $\frac{\pi}{6}$ radians.

b. **[2]**

$$l = r\theta = 50 \times \frac{\pi}{6} = 50 \times \frac{22}{7} \times \frac{1}{6} = \frac{550}{21} \approx 26.19 \text{ cm}$$

In a minute: $40 \times \frac{550}{21} = \frac{22000}{21} \approx 1047.6$ cm, about **10.48 m**.

Marking scheme

- One swing: $l = r\theta \approx 26.19$ cm, with θ in radians.
- A minute: about 1047.6 cm (10.48 m).

c. **Only half right.** The arc is $r\theta$, so doubling θ doubles it (to about 52.38 cm). But $\sin 60^\circ = \frac{\sqrt{3}}{2} \approx 0.87$, while $2\sin 30^\circ = 2 \times \frac{1}{2} = 1$: the sine is not doubled. **[1]**

Marking scheme

- The arc doubles, but $\sin 60^\circ = \frac{\sqrt{3}}{2} \neq 2\sin 30^\circ = 1$.

d. $\theta = \frac{l}{r} = \frac{26.19}{r}$ radians. As $r \to \infty$, $\frac{26.19}{r} \to 0$: the angle heads for **0**. A very long pendulum barely turns to travel the same arc. **[1]**

Marking scheme

- $\theta = \frac{l}{r} \to 0$ as $r \to \infty$.

Question 17 — 6 marks

a. $\overrightarrow{PP'} = \begin{pmatrix}1\\0\end{pmatrix}$ and $\overrightarrow{QQ'} = \begin{pmatrix}3\\0\end{pmatrix}$. They point the same way but their magnitudes are 1 and 3, so **he is not right**: they are like vectors, not equal ones. (In an enlargement, points farther from the center move farther.) **[1]**

Marking scheme

- Not equal: $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}3\\0\end{pmatrix}$ have different magnitudes.

b. **[2]**

$$\overrightarrow{PR} = \begin{pmatrix}4-2\\3-1\end{pmatrix} = \begin{pmatrix}2\\2\end{pmatrix}, \qquad \overrightarrow{P'R'} = \begin{pmatrix}7-3\\5-1\end{pmatrix} = \begin{pmatrix}4\\4\end{pmatrix} = 2\overrightarrow{PR}$$

$|\overrightarrow{PR}| = \sqrt{4+4} = 2\sqrt{2}$ and $|\overrightarrow{P'R'}| = \sqrt{16+16} = 4\sqrt{2}$: the image side is in the same direction and twice as long.

Marking scheme

- $\overrightarrow{PR} = \begin{pmatrix}2\\2\end{pmatrix}$ and $\overrightarrow{P'R'} = \begin{pmatrix}4\\4\end{pmatrix}$.
- Magnitudes $2\sqrt{2}$ and $4\sqrt{2}$: $\overrightarrow{P'R'} = 2\overrightarrow{PR}$.

c. **Scale factor:** $P'Q' = 7 - 3 = 4$ and $PQ = 4 - 2 = 2$, so $k = \frac{4}{2} = 2$ (positive: the image is on the same side of the center and not turned over). **[3]**

Center: with center (a, b) and $k = 2$, $P(x, y) \to P'(2(x-a)+a,\ 2(y-b)+b) = (2x - a,\ 2y - b)$. For P: $2 \times 2 - a = 3$ and $2 \times 1 - b = 1$, so $a = 1, b = 1$. **The center is** $(1, 1)$**.**

Check: $Q(4, 1) \to (8-1, 2-1) = (7, 1) = Q'$ ✓ and $R(4, 3) \to (8-1, 6-1) = (7, 5) = R'$ ✓. The enlargement is $E[(1, 1), 2]$.

Marking scheme

- Scale factor 2.
- Center $(1, 1)$.
- Checked with Q and R (or with lines PP', QQ', RR' meeting at $(1, 1)$).

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