Final Exam Model Paper 3

ANSWER KEY

Full marks 75 · 3 hours

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

A practice paper by Science Playground, made to CDC's Specification Grid 2078, Mathematics, Grades 9–10; not an official CDC or exam board paper.

Question 1 — 6 marks

a. $E = \{2, 4, 6, 8, 10, 12, 14\}$, so $n(E) = 7$. **[1]**

Marking scheme

- $n(E) = 7$.

b. $P = \{2, 3, 5, 7, 11, 13\}$, $S = \{1, 4, 9\}$, $E = \{2, 4, 6, 8, 10, 12, 14\}$. No number is in all three sets, and none is in both P and S; 4 is in S and E, and 2 in P and E. **[1]**

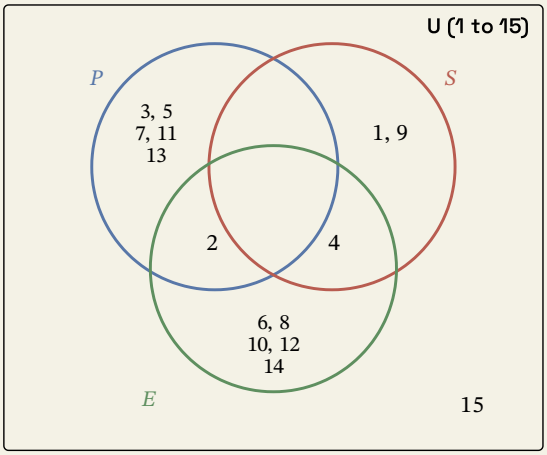


Figure 1 The numbers 1 to 15: only 15 is in none of the three sets, and the regions shared by P and S are empty.

Marking scheme

- A Venn diagram in a rectangle U with every number from 1 to 15 in its right region (15 outside the circles, nothing where P and S overlap).

c. Left side: $P \cap E = \{2\}$, so **[3]**

$$\overline{P \cap E} = \{1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$$

Right side: $\overline{P} = \{1, 4, 6, 8, 9, 10, 12, 14, 15\}$ and $\overline{E} = \{1, 3, 5, 7, 9, 11, 13, 15\}$, so

$$\overline{P} \cup \overline{E} = \{1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$$

Both sides are the same set. **So $\overline{P \cap E} = \overline{P} \cup \overline{E}$.**

Marking scheme

- $P \cap E = \{2\}$ and its complement listed.
- $\overline{P}$ and $\overline{E}$ listed.
- $\overline{P} \cup \overline{E}$ listed, and the two sides said to be equal.

d. $P \cap S = \varnothing$: **P and S are disjoint sets.** The reason: a square number other than 1 is $k \times k$ with $k > 1$, so it has the factor k and isn't prime; and 1 isn't a prime number either. **[1]**

Marking scheme

- Disjoint sets ($P \cap S = \varnothing$), with a reason.

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Question 2 — 5 marks

a. The **social security tax**. **[1]**

Marking scheme

- Social security tax.

b. Yearly income $= 13 \times 110000 = 1430000$. Deposits in the trust $= 12 \times \frac{10}{100} \times 110000 = 12 \times 11000 = 132000$. **[2]**

$$\text{Taxable income} = 1430000 - 132000 = 1298000$$

Rs 12,98,000.

Marking scheme

- The yearly income Rs 14,30,000 and the deposits Rs 1,32,000.
- Taxable income Rs 12,98,000.

c. Rs 12,98,000 = Rs 10,00,000 + Rs 2,98,000. **[2]**

$$\begin{aligned}\text{Tax} &= \tfrac{1}{100} \times 1000000 + \tfrac{10}{100} \times 298000 \\ &= 10000 + 29800 = 39800\end{aligned}$$

Relief: $\frac{10}{100} \times 39800 = 3980$. She pays $39800 - 3980 = 35820$.

Rs 35,820.

Marking scheme

- The tax before relief: Rs 39,800.
- Rs 35,820 after the 10% relief.

Question 3 — 3 marks

a. A **bonus** is money that an organization pays its employees once a year, on top of their salary, as a part of the profit it made that year. **[1]**

Marking scheme

- A part of the year's profit paid to the employees, on top of their salary.

b. Bonus fund $= \frac{10}{100} \times 7200000 = 720000$; each $= 720000 \div 48 =$ **Rs 15,000**. **[1]**

Marking scheme

- Rs 15,000.

c. New profit $= 7200000 - \frac{20}{100} \times 7200000 = 5760000$; bonus fund $= 576000$; each of 50 employees gets $576000 \div 50 = 11520$. **[1]**

Each bonus falls by $15000 - 11520 =$ **Rs 3,480.**

Marking scheme

- Rs 3,480 (each new bonus Rs 11,520).

Question 4 — 5 marks

a. 9 units is less than the minimum 10 units, so the water charge is the minimum, Rs 100. Bill $= 100 + 50 =$ **Rs 150**. **[1]**

Marking scheme

- Rs 150 (the minimum charge with 50% sewerage).

b. 72 units = 20 + 10 + 20 + 22, in the 51–100 block, so the minimum charge is Rs 75. **[2]**

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$$\begin{aligned}\text{Energy charge} &= 20 \times 3 + 10 \times 6.50 + 20 \times 8 + 22 \times 9.50 \\ &= 60 + 65 + 160 + 209 = 494 \\ \text{Bill} &= 75 + 494 = 569\end{aligned}$$

Rs 569.

Marking scheme

- The energy charge block by block: Rs 494.
- The bill: Rs 569.

c. Day: $14 + 39 \times 9 = 14 + 351 = 365$. Night: $21 + 58.50 \times 9 = 21 + 526.50 = 547.50$. **[2]**

$547.50 \div 365 = 1.5$. **Yes, he is right:** the night fare, Rs 547.50, is exactly 1.5 times the day fare, Rs 365, because every night rate is 50% more than the day rate.

Marking scheme

- Both fares: Rs 365 by day and Rs 547.50 at night.
- Yes, the ratio is exactly 1.5.

Question 5 — 5 marks

a. **20 dhur.** **[1]**

Marking scheme

- 20.

b. $s = \dfrac{130 + 140 + 150}{2} = \dfrac{420}{2} = $ **210 ft.** **[1]**

Marking scheme

- $s = 210$ ft.

c. **[2]**

$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{210 \times 80 \times 70 \times 60} \\ &= \sqrt{70560000} = 8400\end{aligned}$$

8,400 ft². In kattha: $8400 \div 3645 = 2.3045\ldots$: 2 kattha, and $0.3045 \times 20 = 6.09$ dhur.

8,400 ft², that is 2 kattha 6.09 dhur.

Marking scheme

- 8,400 ft² from Heron's formula.
- 2 kattha 6.09 dhur.

d. 1 dhur $= 3645 \div 20 = 182.25$ ft², so the neighbor's plot is $2 \times 3645 + 5 \times 182.25 = 7290 + 911.25 = 8201.25$ ft². **[1]**

Rajkumar's field is bigger, by $8400 - 8201.25 = 198.75$ **ft²** (about 1.09 dhur).

Marking scheme

- Rajkumar's, by 198.75 ft².

Question 6 — 4 marks

a. **[1]**

$$A = 4\pi r^2$$

Marking scheme

- $4\pi r^2$.

b. The water rises by as much as the stone's volume, spread over the tin's base. **[3]**

$$\text{Stone} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 3^3 = \frac{792}{7} \approx 113.14\ \text{cm}^3$$

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$$\text{Base of the tin} = \pi R^2 = \frac{22}{7} \times 6^2 = \frac{792}{7} \approx 113.14 \text{ cm}^2$$

$$\text{Rise} = \frac{792/7}{792/7} = 1$$

The water rises by 1 cm. (Both are 36π: $\frac{4}{3}\pi \times 27 = 36\pi$ and $\pi \times 36 = 36\pi$.)

Marking scheme

- The volume of the stone: $36\pi \approx 113.14$ cm³.
- The area of the tin's base: $36\pi \approx 113.14$ cm².
- The rise: 1 cm.

Question 7

4 marks

a. **[2]**

$$\text{Cross-section} = \frac{1}{2} \times 3 \times 0.5 = 0.75 \text{ m}^2$$

$$\text{Volume} = 0.75 \times 1.2 = 0.9 \text{ m}^3$$

0.9 m³ of concrete.

Marking scheme

- The area of the triangular end: 0.75 m².
- The volume: 0.9 m³.

b. Cost $= 0.9 \times 15000 = 13500$. **No, Rs 11,250 is not enough** (Rs 2,250 short). **[2]**

With Rs 11,250 they can buy $11250 \div 15000 = 0.75$ m³. Width $= \dfrac{\text{volume}}{\text{cross-section}} = \dfrac{0.75}{0.75} =$ **1 m.**

Marking scheme

- The cost Rs 13,500, so the money is not enough.
- A width of 1 m.

Question 8

4 marks

a. The **common difference** is the fixed number that each term exceeds the term before it by: $d = t_2 - t_1 = t_3 - t_2 = \cdots$. **[1]**

Marking scheme

- Any term minus the term before it, the same all through.

b. From the 4th row to the 9th row there are 5 steps: **[1]**

$$t_9 - t_4 = 5d \quad \Rightarrow \quad 41 - 26 = 5d \quad \Rightarrow \quad d = 3$$

3 seats more in each row.

Marking scheme

- $d = 3$.

c. $t_4 = a + 3d$: $26 = a + 9$, so $a = 17$. **[2]**

$$t_{20} = a + 19d = 17 + 19 \times 3 = 74$$

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17 seats in the first row and 74 in the 20th.

Marking scheme

- The first row: 17 seats.
- The 20th row: 74 seats.

Question 9 — 7 marks

a. **[1]**

$$a^4 + a^2b^2 + b^4 = (a^2 + ab + b^2)(a^2 - ab + b^2)$$

Marking scheme

- $(a^2 + ab + b^2)(a^2 - ab + b^2)$.

b. First expression: **[3]**

$$27a^3 + 1 = (3a)^3 + 1^3 = (3a + 1)(9a^2 - 3a + 1)$$

Second expression:

$$\begin{aligned} 81a^4 + 9a^2 + 1 &= (9a^2 + 1)^2 - 18a^2 + 9a^2 \\ &= (9a^2 + 1)^2 - (3a)^2 \\ &= (9a^2 + 3a + 1)(9a^2 - 3a + 1) \end{aligned}$$

The factor in both: **HCF** $= 9a^2 - 3a + 1$**.**

Marking scheme

- $27a^3 + 1 = (3a + 1)(9a^2 - 3a + 1)$.
- $81a^4 + 9a^2 + 1 = (9a^2 + 3a + 1)(9a^2 - 3a + 1)$.
- HCF $= 9a^2 - 3a + 1$.

c. Division law, then power law: **[3]**

$$x^{\frac{a-b}{ab}} \times x^{\frac{b-c}{bc}} \times x^{\frac{c-a}{ca}}$$

Each index splits: $\dfrac{a-b}{ab} = \dfrac{1}{b} - \dfrac{1}{a}, \dfrac{b-c}{bc} = \dfrac{1}{c} - \dfrac{1}{b}, \dfrac{c-a}{ca} = \dfrac{1}{a} - \dfrac{1}{c}$.

Multiplication law:

$$x^{\frac{1}{b} - \frac{1}{a} + \frac{1}{c} - \frac{1}{b} + \frac{1}{a} - \frac{1}{c}} = x^0 = 1$$

1.

Marking scheme

- Each bracket as one power: $x^{\frac{a-b}{ab}}$ and so on.
- The indices split, $\frac{1}{b} - \frac{1}{a}$ and so on, or added over the common denominator abc.
- $x^0 = 1$.

Question 10 — 4 marks

a. Let Sujata be x years old and her mother y years old: **[2]**

$$\begin{aligned} y &= 4x \quad \text{(i)} \\ y + 5 &= 3(x + 5) \quad \text{(ii)} \end{aligned}$$

Put (i) in (ii): $4x + 5 = 3x + 15$, so $x = 10$ and $y = 40$.

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Sujata is 10 years old and her mother 40. Check: after 5 years, 45 = 3 × 15 ✓

Marking scheme

- The equations $y = 4x$ and $y + 5 = 3(x + 5)$.
- Sujata 10 years and her mother 40 years.

b. After t years: $40 + t = 2(10 + t)$, so $40 + t = 20 + 2t$ and $t = 20$. **After 20 years**, when they are 30 and 60. **[2]**

No, never again: the difference in their ages stays 30 years, so the ratio keeps falling towards 1 as both grow older (four times now, three times after 5 years, twice after 20).

Marking scheme

- After 20 years (from $40 + t = 2(10 + t)$).
- No, with the reason that the age difference stays 30 years, so the ratio only falls.

Question 11 — 5 marks

a. **Figures:** draw two triangles ABC of different sizes, for example with $AB = 3$ cm, $BC = 4$ cm, $CA = 5$ cm, and $AB = 4$ cm, $BC = 6$ cm, $CA = 7$ cm. **[3]**

Table: measure the sides and add them in pairs:

Figure	$AB + BC$	CA	$BC + CA$	AB	$CA + AB$	BC	Result
I	7 cm	5 cm	9 cm	3 cm	8 cm	4 cm	each sum greater
II	10 cm	7 cm	13 cm	4 cm	11 cm	6 cm	each sum greater

Conclusion: the sum of any two sides of a triangle is greater than the third side.

Marking scheme

- Two triangles of different sizes with their sides measured.
- A table of the three sums and the third sides, with a result for each figure.
- The conclusion in words.

b. In $\triangle ADE$ and $\triangle ABC$: $\angle ADE = \angle ABC$ (corresponding angles, $DE \parallel BC$) and $\angle A$ is common. So $\triangle ADE \sim \triangle ABC$, and their corresponding sides are in the same ratio: **[2]**

$$\frac{AD}{AB} = \frac{DE}{BC} \quad \Rightarrow \quad \frac{4}{4+6} = \frac{5}{BC} \quad \Rightarrow \quad BC = \frac{5 \times 10}{4} = 12.5$$

$BC = 12.5$ **cm.**

Marking scheme

- $\triangle ADE \sim \triangle ABC$, with the equal angles as the reason.
- $BC = 12.5$ cm.

Question 12 — 4 marks

a. **The diagonals of a parallelogram bisect each other:** $AO = OC$ and $BO = OD$. **[1]**

Marking scheme

- They bisect each other.

b. $AO = OC$: $2x + 1 = x + 4$, so $x = 3$. Then $AO = OC = 7$ cm and $AC =$ **14 cm**. **[1]**

Marking scheme

- $x = 3$ and $AC = 14$ cm.

c. Draw $AB = 6.5$ cm. At B, construct $\angle ABX = 60°$ with a compass and cut $BC = 4$ cm on BX. At C, construct $\angle BCY = 120°$ on the same side as A (co-interior with 60°, so $CY \parallel AB$), and cut $CD = 3.5$ cm on CY. Join AD. **[2]**

$ABCD$ **is the trapezium.** See it step by step in Construct it: a trapezium.

Marking scheme

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- AB, the 60∘ angle at B with compass arcs and $BC = 4$ cm.
- $CD \parallel AB$ (120∘ at C), $CD = 3.5$ cm, and the trapezium completed.

Question 13 — 4 marks

a. $\angle OMA = 90°$**.** The line joining the center of a circle and the midpoint of a chord is perpendicular to the chord. **[1]**

Marking scheme

- 90∘, with the theorem.

b. $AM = \frac{48}{2} = 24$ cm. In the right-angled $\triangle OMA$, the radius OA is the hypotenuse: **[2]**

$$OA = \sqrt{OM^2 + AM^2} = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25$$

The radius is 25 cm.

Marking scheme

- $AM = 24$ cm with $\angle OMA = 90°$ used.
- The radius: 25 cm.

c. The distance of CD from the center: $\sqrt{25^2 - 7^2} = \sqrt{576} = 24$ cm. AB **is nearer** (7 cm against 24 cm): the longer chord is nearer the center. **[1]**

Marking scheme

- AB, with the reason (CD is 24 cm from the center, or the longer chord is nearer).

Question 14 — 5 marks

a. Each student's size is written separately, as collected: **raw data, an individual series.** **[1]**

Marking scheme

- Individual series (raw data).

b. **[1]**

Shoe size	Tally	Students (f)
4	|	1
5	||||	4
6	~~||||~~ |||	8
7	~~||||~~	5
8	||	2
		$N = 20$

Marking scheme

- The table with frequencies 1, 4, 8, 5 and 2 (total 20).

c. **[2]**

$$\sum fX = 4 \times 1 + 5 \times 4 + 6 \times 8 + 7 \times 5 + 8 \times 2 = 4 + 20 + 48 + 35 + 16 = 123$$

$$\overline{X} = \frac{123}{20} = 6.15$$

The mean size is 6.15.

Marking scheme

- $\sum fX = 123$.
- Mean $= 6.15$.

d. **The mode, size 6:** it is the size most students wear (8 of 20). The mean, 6.15, isn't a size shoes are made in, and it says nothing about which size is most needed. **[1]**

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Marking scheme

- The mode, size 6, with a reason.

Question 15 — 6 marks

a. **[1]**

$$\text{Empirical probability} = \frac{\text{number of times the event happened}}{\text{total number of trials}}$$

Marking scheme

- Times the event happened ÷ number of trials.

b. Trials: 200 seeds; the event happened 170 times. **[2]**

$$P(\text{germinates}) = \frac{170}{200} = \frac{17}{20} = 0.85$$

$\frac{17}{20}$ (0.85).

Marking scheme

- The values: 170 out of 200.
- $\frac{17}{20}$ (0.85).

c. **[2]**

$$P(\text{does not germinate}) = 1 - \frac{17}{20} = \frac{3}{20}$$

Expected number: $\frac{3}{20} \times 1000 = 150$. **About 150 seeds.**

Marking scheme

- $P(\text{not}) = \frac{3}{20}$ (0.15).
- About 150 seeds.

d. **No.** An empirical probability comes from a test, so it only estimates what to expect: the 1,000 seeds may give a few more or a few fewer than 850, though the number is likely to be near it. **[1]**

Marking scheme

- No, with the reason that an empirical probability gives only an estimate.

Question 16 — 4 marks

a. $\sin 30^\circ = \frac{1}{2}$. **[1]**

Marking scheme

- $\frac{1}{2}$.

b. **[1]**

$$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4} + \frac{3}{4} = \frac{3}{2}$$

$\frac{3}{2}$.

Marking scheme

- $\frac{3}{2}$.

c. **[1]**

$$\sin 30^\circ = \frac{BK}{AK} \quad \Rightarrow \quad BK = 60 \times \frac{1}{2} = 30$$

30 m.

Marking scheme

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- 30 m.

d. $\sin\theta = \dfrac{30\sqrt{3}}{60} = \dfrac{\sqrt{3}}{2} = \sin 60^\circ$, so $\theta = 60^\circ$. **[1]**

Marking scheme

- $60\circ$, from $\sin\theta = \frac{\sqrt{3}}{2}$.

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