Final Exam Model Paper 2

ANSWER KEY

Full marks 75 · 3 hours

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

A practice paper by Science Playground, made to CDC's Specification Grid 2078, Mathematics, Grades 9–10; not an official CDC or exam board paper.

Question 1

6 marks

a. **[1]**

$$n(F \cup V) = n(F) + n(V) - n(F \cap V)$$

Marking scheme

- $n(F \cup V) = n(F) + n(V) - n(F \cap V)$.

b. Football only: $35 - 10 = 25$; volleyball only: $28 - 10 = 18$; both: 10; neither: 7. **[1]**

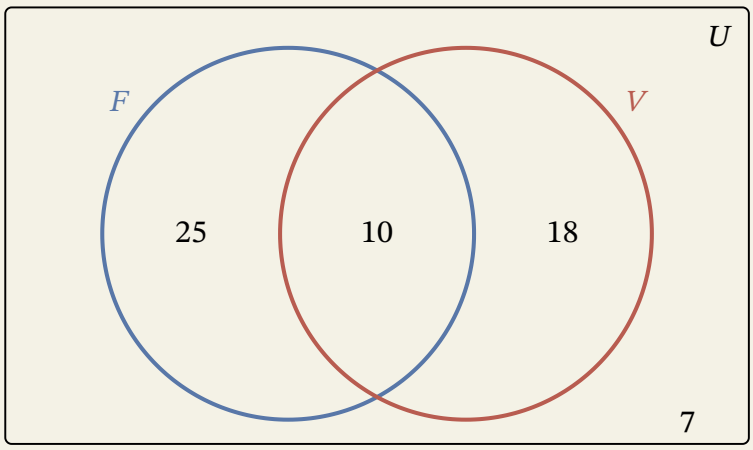


Figure 1 25 like football only, 10 both games, 18 volleyball only and 7 neither.

Marking scheme

- A Venn diagram in a rectangle U with 25, 10, 18 and 7 in their regions.

c. **[3]**

$$n(F \cup V) = n(F) + n(V) - n(F \cap V) = 35 + 28 - 10 = 53$$

53 students like at least one of the games, and 7 like neither:

$$n(U) = n(F \cup V) + n(\overline{F \cup V}) = 53 + 7 = 60$$

60 students were asked. Check from the diagram: $25 + 10 + 18 + 7 = 60$ ✓

Marking scheme

- The values in the formula: $35 + 28 - 10$.
- $n(F \cup V) = 53$.
- $n(U) = 53 + 7 = 60$ students.

d. Only football: 25 students. Volleyball (with or without football): 28 students. **No:** 25 is less than 28, so fewer students like only football than like volleyball. **[1]**

Marking scheme

- No, with the reason: 25 like only football and 28 like volleyball.

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Question 2 — 5 marks

a. **VAT** is a tax on the value added to goods and services at each stage of making and selling them; the buyer pays it on the price, and the seller passes it to the government. In Nepal it is 13%. **[1]**

Marking scheme

- A tax on the value added to goods or services, paid by the buyer on the price and collected by the government (13% in Nepal).

b. **[2]**

$$\text{Discount} = \frac{15}{100} \times 48000 = 7200$$

$$\text{Price after discount} = 48000 - 7200 = 40800$$

Rs 40,800.

Marking scheme

- The discount, Rs 7,200.
- Rs 40,800.

c. VAT is added after the discount: **[2]**

$$\text{VAT} = \frac{13}{100} \times 40800 = 5304$$

$$\text{She pays } 40800 + 5304 = 46104$$

Rs 46,104.

What it means: still Rs 1,896 less than the marked price, as the discount is bigger than the tax.

Marking scheme

- VAT on the price after discount: Rs 5,304.
- Rs 46,104.

Question 3 — 3 marks

a. The **shareholders** of the company, each in proportion to the number of shares they own. **[1]**

Marking scheme

- The shareholders, by the number of shares they hold.

b. Dividend per share $= \dfrac{20000000}{5000000} = 4$: Rs 4 on a share of Rs 100, that is $\frac{4}{100} \times 100\% =$ **4%** of the face value. **[1]**

Marking scheme

- 4% (Rs 4 a share).

c. This year's dividend is $\frac{2}{8} = 25\%$ of the profit. Next year: $\frac{25}{100} \times 100000000 = 25000000$; per share $25000000 \div 5000000 = 5$. **[1]**

Bimala: $1200 \times 5 =$ **Rs 6,000**.

Marking scheme

- Rs 6,000 (Rs 5 a share, from 25% of the new profit).

Question 4 — 5 marks

a. $21 + 58.50 \times 4 = 21 + 234 =$ **Rs 255**. **[1]**

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Marking scheme

- Rs 255.

b. 80,000 liters = 80 units: 56 units at the minimum charge and $80 - 56 = 24$ units more. **[2]**

$$\text{Water charge} = 3960 + 24 \times 71 = 3960 + 1704 = 5664$$
$$\text{Bill} = 5664 + \tfrac{50}{100} \times 5664 = 5664 + 2832 = 8496$$

Rs 8,496.

Marking scheme

- The water charge: Rs 5,664.
- The bill with 50% sewerage: Rs 8,496.

c. Both taxes together make the bill $\text{TC} \times 1.13 \times 1.13 = 1.2769 \times \text{TC}$. **[2]**

$$\text{TC} \le \frac{750}{1.2769} = 587.36\ldots$$

The TC is a whole number of rupees, so at most Rs 587. Calls $= 175 + (587 - 200) =$ **562 calls**.

Check: 562 calls cost $587 \times 1.2769 = 749.54$; one more call would cost $588 \times 1.2769 = 750.82$, over Rs 750.

Marking scheme

- The largest TC: Rs 587 (dividing by 1.2769, not subtracting the taxes).
- 562 calls (the first 175 added back).

Question 5 — 5 marks

a. **[1]**

$$\text{Area of four walls} = 2h(l + b)$$

where l, b and h are the room's length, breadth and height.

Marking scheme

- $2h(l + b)$.

b. $2 \times 3.5 \times (8 + 6) = 7 \times 14 =$ **98 m²**. **[1]**

Marking scheme

- 98 m².

c. Ceiling $= 8 \times 6 = 48$ m². Door $= 2 \times 1.2 = 2.4$ m²; windows $= 3 \times 1.5 \times 1.2 = 5.4$ m². **[2]**

$$\text{Area to paint} = 98 + 48 - 2.4 - 5.4 = 138.2\ \text{m}^2$$

$$\text{Cost} = 138.2 \times 160 = 22112$$

Rs 22,112.

Marking scheme

- The area to paint: 138.2 m².
- Rs 22,112.

d. Two coats: $2 \times 138.2 = 276.4$ m². Tins: $276.4 \div 40 = 6.91$. A part of a tin can't be bought, and 6 tins would cover only 240 m², so **7 tins** must be bought. **[1]**

Marking scheme

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- 7 tins, from $276.4 \div 40 = 6.91$, rounded up.

Question 6 — 4 marks

a. **[1]**

$$V = \frac{2}{3}\pi r^3$$

Marking scheme

- $V = \frac{2}{3}\pi r^3$.

b. **[3]**

$$\text{Pot} = \pi r^2 h = \frac{22}{7} \times 21^2 \times 28 = 38808 \text{ cm}^3$$

$$\text{Bowl} = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 10.5^3 = 2425.5 \text{ cm}^3$$

$$\text{Bowls} = \frac{38808}{2425.5} = 16$$

16 bowls.

Marking scheme

- The volume of the pot: 38,808 cm³.
- The volume of a bowl: 2,425.5 cm³.
- 16 bowls.

Question 7 — 4 marks

a. **[2]**

$$\text{Cross-section} = \frac{1}{2}(1.2 + 0.8) \times 0.6 = 0.6 \text{ m}^2$$

$$\text{Volume} = 0.6 \times 3 = 1.8 \text{ m}^3$$

0.6 m², and 1.8 m³ (1,800 liters) of water.

Marking scheme

- The cross-section: 0.6 m².
- The volume: 1.8 m³ (1,800 liters).

b. Time to fill $= 1800 \div 12 = 150$ minutes $= 2$ hours 30 minutes, so the trough is full at 8:30. **[2]**

No: at 8:15 it is not yet full; it needs 15 minutes more (it holds $135 \times 12 = 1620$ liters then).

Marking scheme

- 150 minutes to fill (2 hours 30 minutes).
- No, it is full only at 8:30.

Question 8 — 4 marks

a. **[1]**

$$t_n = ar^{n-1}$$

where a is the first term and r the common ratio.

Marking scheme

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- $t_n = ar^{n-1}$.

b. $r = \dfrac{t_2}{t_1} = \dfrac{4}{8} = \dfrac{1}{2}$. **[1]**

Marking scheme

- $r = \frac{1}{2}$.

c. $a = 8, r = \frac{1}{2}, t_n = 0.25 = \frac{1}{4}$: **[2]**

$$8 \times \left(\tfrac{1}{2}\right)^{n-1} = \tfrac{1}{4}$$
$$\left(\tfrac{1}{2}\right)^{n-1} = \tfrac{1}{32} = \left(\tfrac{1}{2}\right)^5$$

$n - 1 = 5, n = 6$. **After the 6th bounce.**

Marking scheme

- The values in the formula: $8 \times \left(\frac{1}{2}\right)^{n-1} = \frac{1}{4}$.
- The 6th bounce.

Question 9 — 7 marks

a. **[1]**

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

Marking scheme

- $a^3 - 3a^2b + 3ab^2 - b^3$.

b. First expression: **[3]**

$$27y^3 - 8 = (3y)^3 - 2^3 = (3y-2)(9y^2 + 6y + 4)$$

Second expression:

$$\begin{aligned} 81y^4 + 36y^2 + 16 &= (9y^2+4)^2 - 72y^2 + 36y^2 \\ &= (9y^2+4)^2 - (6y)^2 \\ &= (9y^2 + 6y + 4)(9y^2 - 6y + 4) \end{aligned}$$

The LCM takes every factor, the shared one once:
LCM $= (3y-2)(9y^2+6y+4)(9y^2-6y+4)$**.**

Marking scheme

- $27y^3 - 8 = (3y-2)(9y^2+6y+4)$.
- $81y^4 + 36y^2 + 16 = (9y^2+6y+4)(9y^2-6y+4)$.
- LCM $= (3y-2)(9y^2+6y+4)(9y^2-6y+4)$.

c. Multiplication law: $3^{n+2} = 3^n \times 3^2$ and $3^{n+1} = 3^n \times 3$: **[3]**

$$\frac{3^n \times 9 - 3^n}{3^n \times 3 + 3^n}$$

Take out the common factor 3^n:

$$\frac{3^n(9-1)}{3^n(3+1)} = \frac{8}{4}$$

$= \mathbf{2}$.

Marking scheme

- $3^{n+2} = 3^n \times 3^2$ and $3^{n+1} = 3^n \times 3$.

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- $\dfrac{3^n(9-1)}{3^n(3+1)}$.
- 2.

Question 10 — 4 marks

a. Let there be x hens and y goats: **[2]**

$$x + y = 46 \quad \text{(i)}$$
$$2x + 4y = 128 \quad \text{(ii)}$$

(ii) − 2 × (i): $2y = 128 - 92 = 36$, so $y = 18$ and $x = 46 - 18 = 28$.

28 hens and 18 goats. Check: $2 \times 28 + 4 \times 18 = 56 + 72 = 128$ ✓

Marking scheme

- The equations $x + y = 46$ and $2x + 4y = 128$.
- 28 hens and 18 goats.

b. Selling k hens takes away k heads and $2k$ legs: **[2]**

$$128 - 2k = 3(46 - k)$$
$$128 - 2k = 138 - 3k$$
$$k = 10$$

He sold 10 hens. Check: 18 hens and 18 goats have 36 heads and $36 + 72 = 108 = 3 \times 36$ legs ✓

Marking scheme

- The equation $128 - 2k = 3(46 - k)$.
- 10 hens.

Question 11 — 5 marks

a. **Figures:** draw two triangles ABC with sides of different lengths, for example $BC = 6$ cm, $CA = 5$ cm, $AB = 4$ cm, and $BC = 8$ cm, $CA = 6$ cm, $AB = 5$ cm. **[3]**

Table: measure the angles opposite the longest and the shortest side:

Figure	Longest side BC	$\angle A$ (opposite BC)	Shortest side AB	$\angle C$ (opposite AB)	Result
I	6 cm	83°	4 cm	41°	$\angle A > \angle C$
II	8 cm	93°	5 cm	39°	$\angle A > \angle C$

(Measured angles may differ by a degree or so.)

Conclusion: in a triangle, the angle opposite the longer side is greater than the angle opposite the shorter side.

Marking scheme

- Two triangles of different sizes with their sides measured.
- A table of the sides and the angles opposite them, with a result for each figure.
- The conclusion in words.

b. The exterior angle BAE equals the two opposite interior angles: $x + 2x = 120°$, so $x = 40°$, and $\angle ABC = 40°$. $\angle BAC = 180° - 120° = 60°$ (a linear pair). **[2]**

$\angle BAC : \angle ABC = 60 : 40 =$ **3 : 2**.

Marking scheme

- $x = 40°$ and $\angle BAC = 60°$, with reasons.
- The ratio 3 : 2.

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Question 12 — 4 marks

a. **The diagonals of a rectangle are equal and bisect each other:** $PR = QS$, and $PO = OR = QO = OS$. **[1]**

Marking scheme

- They are equal and bisect each other.

b. $PR = QS$: $4y - 2 = 2y + 8$, so $2y = 10$ and $y = 5$. Then $PR = 18$ cm, and $OP = \frac{1}{2}PR =$ **9 cm.** **[1]**

Marking scheme

- $y = 5$ and $OP = 9$ cm.

c. Draw a rough sketch first. Draw $AB = 5.5$ cm. With center A and radius 6.5 cm, and with center B and radius 4.5 cm, draw arcs that cut at C; join AC and BC. On the other side of AC from B, draw an arc of radius 4 cm from A and one of radius 5 cm from C; they cut at D. Join AD and CD. **[2]**

$ABCD$ **is the quadrilateral.** See it step by step in Construct it: a quadrilateral.

Marking scheme

- $\triangle ABC$ drawn from its three sides with the arcs showing.
- D found on the other side of AC by arcs of 4 cm and 5 cm, and the quadrilateral completed.

Question 13 — 4 marks

a. $OM = ON$**:** equal chords of a circle are equidistant from the center. **[1]**

Marking scheme

- $OM = ON$, with the theorem.

b. The perpendicular from the center bisects the chord, so $MB = 8$ cm. Join OB, a radius: in the right-angled $\triangle OMB$, **[2]**

$$OM = \sqrt{OB^2 - MB^2} = \sqrt{10^2 - 8^2} = \sqrt{36} = 6$$

$OM = 6$ **cm.**

Marking scheme

- $MB = 8$ cm, half the chord.
- $OM = 6$ cm.

c. **16 cm.** EF is as far from the center as AB (6 cm), and chords which are equidistant from the center of a circle are equal, so $EF = AB = 16$ cm. (Or: half of EF is $\sqrt{10^2 - 6^2} = 8$ cm.) **[1]**

Marking scheme

- 16 cm, with the reason (equidistant chords are equal, or by Pythagoras' rule).

Question 14 — 5 marks

a. Each age is given with its frequency: a **discrete series**. **[1]**

Marking scheme

- Discrete series.

b. **[1]**

Age	f	cf
14	3	3
15	5	8
16	7	15
17	6	21
18	4	25

Marking scheme

- The cf column 3, 8, 15, 21, 25.

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c. [2]

$$\text{Median} = \left(\frac{N+1}{2}\right)\text{th item} = \left(\frac{25+1}{2}\right)\text{th item} = 13\text{th item}$$

In the cf column, 15 is the first number of 13 or more, so the 13th player is 16. **The median age is 16 years.**

Marking scheme

- The 13th item.
- Median = 16 years.

d. $Q_1 = \left(\frac{N+1}{4}\right)$th item $= 6.5$th item: the 6th and 7th players are both 15 (the cf 8 covers items 4 to 8), so $Q_1 = 15$. [1]

$Q_3 = \left(\frac{3(N+1)}{4}\right)$th item $= 19.5$th item: the 19th and 20th are both 17 (items 16 to 21), so $Q_3 = 17$.

The organizers are right: the middle half of the players lie between $Q_1 = 15$ and $Q_3 = 17$ years.

Marking scheme

- $Q_1 = 15$ and $Q_3 = 17$, so the organizers are right.

Question 15 — 6 marks

a. **0.** (For example, the probability of getting a 7.) [1]

Marking scheme

- 0.

b. $S = \{1, 2, 3, 4, 5, 6\}, n(S) = 6$. The prime numbers are $\{2, 3, 5\}$, so $n(E) = 3$. [2]

$$P(\text{prime}) = \frac{3}{6} = \frac{1}{2}$$

$\frac{1}{2}$. (1 is not a prime number.)

Marking scheme

- $n(S) = 6$ and $n(E) = 3$.
- $P = \frac{1}{2}$.

c. The multiples of 3 are $\{3, 6\}$: $P(\text{multiple of } 3) = \frac{2}{6} = \frac{1}{3}$. [2]

$$P(\text{not a multiple of } 3) = 1 - \frac{1}{3} = \frac{2}{3}$$

$\frac{2}{3}$.

Marking scheme

- $P(\text{multiple of } 3) = \frac{1}{3}$.
- $P(\text{not}) = \frac{2}{3}$.

d. Greater than 4: $\{5, 6\}$, probability $\frac{2}{6} = \frac{1}{3}$. Even: $\{2, 4, 6\}$, probability $\frac{3}{6} = \frac{1}{2}$. **No:** an even number is more likely, as it has more favorable outcomes. [1]

Marking scheme

- No, with the two probabilities $\frac{1}{3}$ and $\frac{1}{2}$.

Question 16 — 4 marks

a. PQ is opposite θ (the perpendicular) and PR is the hypotenuse: $\sin\theta = \frac{p}{h} = \frac{PQ}{PR}$. [1]

Marking scheme

- $\sin\theta = \frac{PQ}{PR}$.

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b. $PR = \sqrt{PQ^2 + QR^2} = \sqrt{64 + 225} = \sqrt{289} = 17$ cm. **[1]**

Marking scheme

- $PR = 17$ cm.

c. $\cos\theta = \frac{QR}{PR} = \frac{15}{17}$ and $\sin\theta = \frac{8}{17}$: **[1]**

$$\frac{1 - \frac{15}{17}}{\frac{8}{17}} = \frac{\frac{2}{17}}{\frac{8}{17}} = \frac{2}{8} = \frac{1}{4}$$

$\frac{1}{4}$.

Marking scheme

- $\frac{1}{4}$.

d. $\tan 30^\circ = \frac{PQ}{QR}$, so $PQ = 15 \times \frac{1}{\sqrt{3}} = \frac{15}{\sqrt{3}} = 5\sqrt{3} \approx 8.66$. $5\sqrt{3}$ cm, about 8.66 cm. **[1]**

Marking scheme

- $PQ = 5\sqrt{3}$ cm (about 8.66 cm).

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