Final Exam Model Paper 1

ANSWER KEY

Full marks 75 · 3 hours

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

A practice paper by Science Playground, made to CDC's Specification Grid 2078, Mathematics, Grades 9–10; not an official CDC or exam board paper.

Question 1 — 6 marks

a. $C = \{1, 2, 3, 6, 9, 18\}$, so $n(C) = 6$. **[1]**

Marking scheme

- $n(C) = 6$.

b. $A = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}, B = \{3, 6, 9, 12, 15, 18\}, C = \{1, 2, 3, 6, 9, 18\}$. Place the numbers in all three sets first (6 and 18), then those in two (12 in A and B; 3 and 9 in B and C; 2 in A and C), then the rest. **[1]**

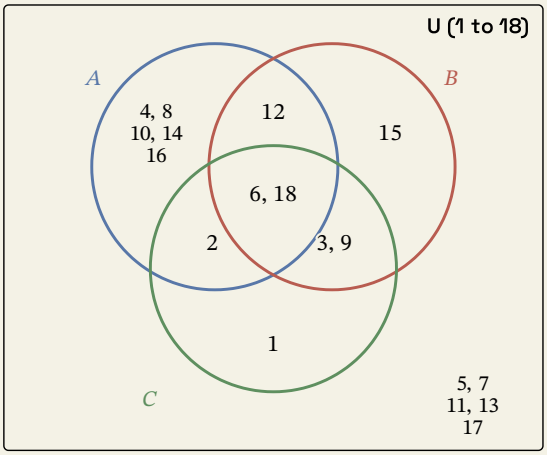


Figure 1 The numbers 1 to 18: 6 and 18 are in all three sets; 5, 7, 11, 13 and 17 are in none.

Marking scheme

- A Venn diagram in a rectangle U with every number from 1 to 18 in its right region (5, 7, 11, 13 and 17 outside the circles).

c. Left side: $B \cup C = \{1, 2, 3, 6, 9, 12, 15, 18\}$, so **[3]**

$$A \cap (B \cup C) = \{2, 6, 12, 18\}$$

Right side: $A \cap B = \{6, 12, 18\}$ and $A \cap C = \{2, 6, 18\}$, so

$$(A \cap B) \cup (A \cap C) = \{2, 6, 12, 18\}$$

Both sides are the same set. **So** $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$**.**

Marking scheme

- $B \cup C = \{1, 2, 3, 6, 9, 12, 15, 18\}$ and $A \cap (B \cup C) = \{2, 6, 12, 18\}$.
- $A \cap B = \{6, 12, 18\}$ and $A \cap C = \{2, 6, 18\}$.
- $(A \cap B) \cup (A \cap C) = \{2, 6, 12, 18\}$, and the two sides said to be equal.

d. $A \cap B = \{6, 12, 18\}$, and the multiples of 6 from 1 to 18 are also $\{6, 12, 18\}$. **They are equal sets.** **[1]**
The reason: a number that is even (divisible by 2) and a multiple of 3 is divisible by $2 \times 3 = 6$, and every multiple of 6 is both.

Marking scheme

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- Equal sets, with the reason (both are $\{6, 12, 18\}$, or divisible by 2 and by 3 means divisible by 6).

Question 2 — 5 marks

a. **Taxable income** is the income of a year on which income tax is charged: the year's income less the deductions the law allows (such as provident fund deposits and insurance premiums). **[1]**

Marking scheme

- The year's income less the deductions allowed by law, on which the tax is charged.

b. **[2]**

$$\text{Yearly income} = 13 \times 85000 = 1105000$$

Rs 11,05,000.

Marking scheme

- 13 months taken: 13×85000.
- Rs 11,05,000.

c. Taxable income $= 1105000 - 30000 = 1075000$: Rs 10,75,000 = Rs 10,00,000 + Rs 75,000. **[2]**

$$\begin{aligned}\text{Tax} &= \tfrac{1}{100} \times 1000000 + \tfrac{10}{100} \times 75000 \\ &= 10000 + 7500 \\ &= 17500\end{aligned}$$

He pays Rs 17,500.

What it means: about 1.6% of his income, though his last Rs 75,000 pays 10%.

Marking scheme

- Taxable income Rs 10,75,000 (the premium taken off the income).
- Rs 17,500, with 1% on the first Rs 10,00,000 and 10% on the rest.

Question 3 — 3 marks

a. A **commission** is the money paid to an agent for buying or selling goods for someone else, usually a percentage of the value of what is sold (or bought). **[1]**

Marking scheme

- Money paid to an agent for selling (or buying) for someone else, as a percentage of the sale.

b. **[1]**

$$\text{Commission} = \frac{4}{100} \times 360000 = 14400$$

Rs 14,400.

Marking scheme

- Rs 14,400.

c. New scheme: $\frac{2}{100} \times 200000 + \frac{6}{100} \times 160000 = 4000 + 9600 = 13600$. **[1]**

The old 4% scheme pays her more, by $14400 - 13600 =$ **Rs 800.** (The new scheme pays more only on bigger sales: above Rs 4,00,000 a month.)

Marking scheme

- The new scheme gives Rs 13,600, so the old scheme pays Rs 800 more.

Question 4 — 5 marks

a. $5376 - 5238 =$ **138 units.** **[1]**

Marking scheme

- 138 units.

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b. 138 units is in the 101–250 block, so the minimum charge is Rs 125. Cut 138 units at the blocks: $138 = 20 + 10 + 20 + 88$. **[2]**

$$\begin{aligned}\text{Energy charge} &= 20 \times 4 + 10 \times 6.50 + 20 \times 8 + 88 \times 9.50 \\ &= 80 + 65 + 160 + 836 = 1141 \\ \text{Bill} &= 125 + 1141 = 1266\end{aligned}$$

Rs 1,266.

Marking scheme

- The energy charge block by block: Rs 1,141.
- The bill with the minimum charge of Rs 125: Rs 1,266.

c. The first 50 units cost $80 + 65 + 160 = 305$ in energy charge, and each unit after that Rs 9.50. **[2]**

Up to 100 units the minimum charge is Rs 100: the bill for 100 units is $100 + 305 + 50 \times 9.50 = 880$, under Rs 1,000. So they can use more than 100 units, and then the minimum charge is Rs 125. For u units:

$$\begin{aligned}125 + 305 + 9.50(u - 50) &\le 1000 \\ 9.50(u - 50) &\le 570 \\ u - 50 &\le 60\end{aligned}$$

At most 110 units (the bill is then exactly Rs 1,000; 111 units would cost Rs 1,009.50). That is 28 units fewer than this month.

Marking scheme

- The bill above 100 units set up with the minimum charge Rs 125 and Rs 305 for the first 50 units (or the bill for 100 units, Rs 880, found).
- At most 110 units.

Question 5

5 marks

a. **[1]**

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \quad \text{where } s = \frac{a+b+c}{2}$$

(Heron's formula.)

Marking scheme

- $\sqrt{s(s-a)(s-b)(s-c)}$ with $s = \frac{a+b+c}{2}$.

b. **[1]**

$$s = \frac{104 + 112 + 120}{2} = \frac{336}{2} = 168$$

168 ft.

Marking scheme

- $s = 168$ ft.

c. **[2]**

$$\begin{aligned}\text{Area} &= \sqrt{168 \times (168 - 104) \times (168 - 112) \times (168 - 120)} \\ &= \sqrt{168 \times 64 \times 56 \times 48} \\ &= \sqrt{28901376} = 5376\end{aligned}$$

5,376 ft².

Marking scheme

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- The values in Heron's formula: $\sqrt{168 \times 64 \times 56 \times 48}$.
- 5,376 ft².

d. A ropani is 5,476 ft², and the plot is 5,376 ft². **No, he is not right:** the plot is $5476 - 5376 = 100$ ft² short of a ropani (about 1 paisa). **[1]**

Marking scheme

- No, with the reason: 5,376 ft² is 100 ft² less than a ropani.

Question 6 — 4 marks

a. **[1]**

$$V = \pi r^2 h$$

where r is the radius of the base and h the height.

Marking scheme

- $V = \pi r^2 h$.

b. Curved surface area $= 2\pi rh$: **[3]**

$$2 \times \frac{22}{7} \times 0.7 \times h = 6.6$$
$$4.4h = 6.6$$
$$h = 1.5$$

The tank is 1.5 m high.

$$V = \frac{22}{7} \times 0.7^2 \times 1.5 = \frac{22}{7} \times 0.49 \times 1.5 = 2.31 \text{ m}^3$$

1 m³ = 1,000 liters, so the tank holds **2,310 liters**.

Days: $2310 \div 330 =$ **7 days**.

Marking scheme

- The height from the curved surface: $h = 1.5$ m.
- The volume 2.31 m³, or 2,310 liters.
- 7 days.

Question 7 — 4 marks

a. The height of an end from its apex to the middle of the 6 m base: $\sqrt{5^2 - 3^2} = \sqrt{16} = 4$ m. **[2]**

One end $= \frac{1}{2} \times 6 \times 4 = 12$ m²; two ends $= 2 \times 12 =$ **24 m²**.

(By Heron's formula: $s = 8$, $\sqrt{8 \times 3 \times 3 \times 2} = \sqrt{144} = 12$ m².)

Marking scheme

- The area of one end: 12 m².
- 24 m² for the two ends.

b. Volume of a prism = area of its cross-section × length. The cross-section is the 12 m² end: **[2]**

$$12 \times L = 300 \quad \Rightarrow \quad L = 25$$

The new roof space is 25 m long, 15 m longer than the old one (which holds only $12 \times 10 = 120$ m³).

Marking scheme

- The length from $12 \times L = 300$: 25 m.
- 15 m longer.

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Question 8 — 4 marks

a. **[1]**

$$t_n = a + (n-1)d$$

where a is the first term and d the common difference.

Marking scheme

- $t_n = a + (n-1)d$.

b. $d = t_2 - t_1 = 65 - 50 =$ **Rs 15** (she puts in Rs 15 more each week). **[1]**

Marking scheme

- $d = 15$.

c. $a = 50, d = 15, t_n = 350$: **[2]**

$$\begin{aligned} 350 &= 50 + (n-1) \times 15 \\ 15(n-1) &= 300 \\ n - 1 &= 20 \end{aligned}$$

$n = 21$. **In the 21st week.**

Marking scheme

- The values in the formula: $350 = 50 + (n-1) \times 15$.
- The 21st week.

Question 9 — 7 marks

a. **[1]**

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Marking scheme

- $(a+b)(a^2 - ab + b^2)$.

b. First expression: **[3]**

$$8a^3 + 27 = (2a)^3 + 3^3 = (2a+3)(4a^2 - 6a + 9)$$

Second expression:

$$\begin{aligned} 16a^4 + 36a^2 + 81 &= (4a^2+9)^2 - 72a^2 + 36a^2 \\ &= (4a^2+9)^2 - (6a)^2 \\ &= (4a^2 + 6a + 9)(4a^2 - 6a + 9) \end{aligned}$$

The factor in both: **HCF** $= 4a^2 - 6a + 9$**.**

Marking scheme

- $8a^3 + 27 = (2a+3)(4a^2 - 6a + 9)$.
- $16a^4 + 36a^2 + 81 = (4a^2 + 6a + 9)(4a^2 - 6a + 9)$.
- HCF $= 4a^2 - 6a + 9$.

c. Division law in each bracket: $\dfrac{x^a}{x^{-b}} = x^{a-(-b)} = x^{a+b}$, and so on: **[3]**

$$(x^{a+b})^{a-b} \times (x^{b+c})^{b-c} \times (x^{c+a})^{c-a}$$

Power law, with $(a+b)(a-b) = a^2 - b^2$:

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$$x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$$

Multiplication law:

$$x^{a^2-b^2+b^2-c^2+c^2-a^2} = x^0 = 1$$

1.

Marking scheme

- Each bracket as one power: x^{a+b}, x^{b+c}, x^{c+a}.
- The power law: $x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$.
- $x^0 = 1$.

Question 10 — 4 marks

a. Let an adult's fare be Rs x and a child's Rs y: **[2]**

$$3x + 2y = 4700 \quad \text{(i)}$$
$$2x + 3y = 4050 \quad \text{(ii)}$$

Multiply (i) by 3 and (ii) by 2, and subtract:

$$9x + 6y = 14100$$
$$4x + 6y = 8100$$
$$5x = 6000$$

$x = 1200$. From (i): $3600 + 2y = 4700$, so $y = 550$.

An adult pays Rs 1,200 and a child Rs 550. Check (ii): $2400 + 1650 = 4050$ ✓

Marking scheme

- The two equations $3x + 2y = 4700$ and $2x + 3y = 4050$.
- Adult Rs 1,200 and child Rs 550.

b. New fares: adult $1200 + \frac{10}{100} \times 1200 = 1320$; child $550 + 50 = 600$. **[2]**

For 2 adults and 2 children: $2 \times 1320 + 2 \times 600 = 2640 + 1200 = 3840$.

Yes, Rs 4,000 is enough: the tickets cost Rs 3,840, and Rs 160 is left.

Marking scheme

- The new fares: Rs 1,320 and Rs 600.
- Yes, with the cost Rs 3,840 (Rs 160 left).

Question 11 — 5 marks

a. **Figures:** draw two isosceles triangles ABC with $AB = AC$, one small and one large (for example $AB = AC = 4$ cm, $BC = 5$ cm, and $AB = AC = 6$ cm, $BC = 4$ cm). In each, draw the perpendicular bisector of the base BC with a compass: it meets BC at D and goes through A. Join AD. **[3]**

Table: measure $\angle BAD$ and $\angle CAD$, and check that D is the middle of BC:

Figure	BD	DC	$\angle BAD$	$\angle CAD$	Result
I	2.5 cm	2.5 cm	39°	39°	$\angle BAD = \angle CAD$
II	2 cm	2 cm	19.5°	19.5°	$\angle BAD = \angle CAD$

(Measured angles may differ by a degree or so; the two in each row come out equal.)

Conclusion: the perpendicular bisector of the base of an isosceles triangle passes through the vertex and bisects the vertical angle.

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Marking scheme

- Two isosceles triangles of different sizes, each with the perpendicular bisector of its base drawn through the vertex.
- A table of the measurements (BD, DC, $\angle BAD$, $\angle CAD$) with a result for each figure.
- The conclusion in words.

b. $\angle PRQ = 180^\circ - 115^\circ = 65^\circ$ (a linear pair). $PQ = PR$, so $\angle PQR = \angle PRQ = 65^\circ$ (base angles of an isosceles triangle). The exterior angle PRS equals the two opposite interior angles: $\angle QPR = 115^\circ - 65^\circ = 50^\circ$. **[2]**

$\angle QPR : \angle PQR = 50 : 65 =$ **10 : 13**.

Marking scheme

- $\angle PQR = 65^\circ$ and $\angle QPR = 50^\circ$, with reasons.
- The ratio 10 : 13.

Question 12 — 4 marks

a. **The opposite angles of a parallelogram are equal:** $\angle A = \angle C$ and $\angle B = \angle D$. **[1]**

Marking scheme

- Opposite angles are equal.

b. $\angle A = \angle C$: **[1]**

$$3x + 10 = 5x - 30 \quad \Rightarrow \quad 2x = 40 \quad \Rightarrow \quad x = 20$$

$x = 20$, so $\angle A = \angle C = 70^\circ$, and $\angle B = 180^\circ - 70^\circ =$ **110°** (co-interior angles between $AD \parallel BC$).

Marking scheme

- $x = 20$ and $\angle B = 110^\circ$.

c. Draw $PQ = 5$ cm. At P, construct $\angle QPX = 60^\circ$ with a compass and cut $PS = 5$ cm on PX. With centers Q and S and radius 5 cm, draw two arcs that cut each other at R. Join QR and SR. **[2]**

$PQRS$ **is the rhombus** (all four sides 5 cm). Check: the diagonal QS is 5 cm too, since $\triangle PQS$ is equilateral. See it step by step in Construct it: a rhombus.

Marking scheme

- $PQ = 5$ cm and the 60° angle at P drawn with compass arcs, $PS = 5$ cm.
- R found by arcs of 5 cm from Q and S, and the rhombus completed.

Question 13 — 4 marks

a. $AM = MB$**.** The perpendicular drawn from the center of a circle to a chord bisects the chord. **[1]**

Marking scheme

- $AM = MB$, with the theorem (the perpendicular from the center bisects the chord).

b. $AM = MB = \frac{40}{2} = 20$ cm. In the right-angled $\triangle OMB$, $OB = 25$ cm is the hypotenuse: **[2]**

$$OM = \sqrt{OB^2 - MB^2} = \sqrt{25^2 - 20^2} = \sqrt{625 - 400} = \sqrt{225} = 15$$

$OM = 15$ **cm.**

Marking scheme

- $MB = 20$ cm, half the chord.
- $OM = 15$ cm.

c. The deepest point is at the bottom of the pipe, on the line OM produced. From the center to the bottom is a radius, 25 cm, and the water's surface is 15 cm below the center. **The water is** $25 - 15 = 10$ **cm deep.** **[1]**

Marking scheme

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- 10 cm (the radius 25 cm less $OM = 15$ cm).

Question 14 — 5 marks

a. Each value (number of books) is given with its frequency: a **discrete series**. **[1]**

Marking scheme

- Discrete series.

b. **[1]**

Books read	Students (f)	cf
0	3	3
1	5	8
2	8	16
3	7	23
4	4	27
5	3	30

Marking scheme

- The cf column 3, 8, 16, 23, 27, 30.

c. **[2]**

X	f	fX
0	3	0
1	5	5
2	8	16
3	7	21
4	4	16
5	3	15
	$N = 30$	$\sum fX = 73$

$$\overline{X} = \frac{\sum fX}{N} = \frac{73}{30} \approx 2.43$$

About 2.43 books.

Marking scheme

- $\sum fX = 73$.
- The mean $\frac{73}{30} \approx 2.43$ books.

d. More than 2.43 books means 3, 4 or 5 books: $7 + 4 + 3 = 14$ students. Half of 30 is 15. **No, she is not right:** only 14 students, fewer than half, read more than the mean. **[1]**

Marking scheme

- No, with the count: 14 students read more than 2.43 books, fewer than 15.

Question 15 — 6 marks

a. **1.** (For example, the probability that the bead is red, blue or green.) **[1]**

Marking scheme

- 1.

b. $n(S) = 20$ and $n(\text{blue}) = 8$. **[2]**

$$P(\text{blue}) = \frac{8}{20} = \frac{2}{5}$$

$\frac{2}{5}$ (0.4).

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Marking scheme

- $n(S) = 20$ and $n(E) = 8$.
- $P(\text{blue}) = \frac{2}{5}$.

c. **[2]**

$$P(\text{green}) = \frac{7}{20}, \qquad P(\text{not green}) = 1 - \frac{7}{20} = \frac{13}{20}$$

$\frac{13}{20}$ (the 5 red and 8 blue beads: $\frac{13}{20}$ ✓).

Marking scheme

- $P(\text{green}) = \frac{7}{20}$.
- $P(\text{not green}) = \frac{13}{20}$.

d. Add x red beads: **[1]**

$$\frac{5+x}{20+x} = \frac{1}{2} \quad \Rightarrow \quad 10 + 2x = 20 + x \quad \Rightarrow \quad x = 10$$

10 red beads (then 15 of the 30 beads are red).

Marking scheme

- 10 red beads.

Question 16 — 4 marks

a. For θ, BC is the base and AC the hypotenuse: $\frac{BC}{AC} = \frac{b}{h} = \cos\theta$. **[1]**

Marking scheme

- $\cos\theta$.

b. AB is the perpendicular for θ: $\tan\theta = \frac{p}{b} = \frac{AB}{BC} = \frac{12}{5}$. **[1]**

Marking scheme

- $\tan\theta = \frac{12}{5}$.

c. **[1]**

$$\sin 60^\circ = \frac{AB}{AC} \quad \Rightarrow \quad AB = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} \approx 17.32$$

$10\sqrt{3}$ **ft, about 17.32 ft.**

Marking scheme

- $10\sqrt{3}$ ft (about 17.32 ft).

d. $AC = 2BC$, so $\cos\theta = \frac{BC}{AC} = \frac{1}{2} = \cos 60^\circ$. $\theta = 60^\circ$. **[1]**

Marking scheme

- $\theta = 60^\circ$, from $\cos\theta = \frac{1}{2}$.

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