Unit 19 · Transformation

HARDER PRACTICE QUESTIONS

Full marks 26

Write your answers on separate paper. Show your working, and use graph paper wherever it helps. Rotations are about the origin O; anticlockwise is positive.

1 Two moves, one move [3]

$P(3, -2)$ is reflected in the X-axis, and its image is reflected in the Y-axis.

i. Find the two images.
ii. Which single rotation does the same?
iii. Show that this works for any point (x, y).

2 Work backwards [3]

i. A rotation through $+90°$ takes a point to $(-3, 4)$. Where did it start?
ii. A point is translated by $(2, -3)$, then reflected in the X-axis, and ends at $(5, 1)$. Where did it start?

3 Find the move [3]

Triangle $A(1, 2)$, $B(4, 2)$, $C(1, 6)$ has the image $A'(-2, 1)$, $B'(-2, 4)$, $C'(-6, 1)$. Which single transformation is this? Find the image of $D(5, -3)$ under the same transformation.

4 Does the order matter? [3]

Take $P(1, 3)$.

i. Translate it by $(2, 0)$, then reflect the image in the Y-axis.
ii. Now do the same two moves the other way round.
iii. Does the order matter? Find a translation for which it wouldn't matter with this reflection.

5 Which points stay still? [3]

Which points don't move at all under each of these? Give a reason.

i. Reflection in the X-axis.
ii. Rotation through $+90°$ about O.
iii. Translation by $(3, 1)$.

6 Spot the error [3]

Each line has a mistake. Find it and correct it.

i. Rotation through $-90°$ takes $(2, 5)$ to $(-5, 2)$.
ii. Reflection in the Y-axis takes $(3, -4)$ to $(3, 4)$.
iii. Translation 3 units left takes $(2, 1)$ to $(5, 1)$.

7 In a real setting [4]

A straight river runs along the X-axis of a map, with 1 unit for 1 km. Two villages lie on the same side of it, at $A(1, 3)$ and $B(7, 5)$. One pump on the river will send water by straight pipes to both villages. The pipe is shortest when the pump is at T, on the straight line from A to the reflection of B. (The map is made up for the problem.)

i. Find B', the image of B in the X-axis.
ii. Why is $TB = TB'$ for any point T on the river?
iii. Find the shortest total length of pipe.
iv. On graph paper, draw the line AB' and read where T is.

8 Think [2]

The corners of a triangle are labeled A, B, C going clockwise. After a reflection, do they go clockwise or anticlockwise? After a rotation? After a translation? What does this say about how you could tell a reflection from the others?

9 Make your own [2]

Find three different transformations, one after the other, that bring every point back to where it started. Test them on $(2, 3)$, then show that they work for (x, y).