Unit 15 · Congruency and Similarity

HARDER PRACTICE QUESTIONS · ANSWER KEY

Full marks 27

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 A proof with a shared side

ANSWER A · 3

In $\triangle ABC$ and $\triangle ADC$:
$AB = AD$ (given); $CB = CD$ (given); $AC = AC$ (common side).
So $\triangle ABC \cong \triangle ADC$ by **SSS**.
$\angle ABC$ and $\angle ADC$ are corresponding angles of congruent triangles, so they are equal.

MARKING SCHEME

- ✓ The three pairs of sides, with AC common.
- ✓ SSS named.
- ✓ The angles are corresponding angles of congruent triangles.

2 Why the height cuts the base in half

ANSWER HA · 3

In $\triangle ABD$ and $\triangle ACD$:
$\angle ADB = \angle ADC = 90°$ (given); $AB = AC$ (hypotenuses, given); $AD = AD$ (common side).
So the triangles are congruent by **RHS**, and $BD = DC$ (corresponding sides).
RHS fits because we know a right angle, the hypotenuse and a side. We don't know any third side, so SSS can't be used, and the equal sides aren't on either side of a known angle, so SAS can't.

MARKING SCHEME

- ✓ The right angles, the hypotenuses and the common side.
- ✓ RHS.
- ✓ $BD = DC$, as corresponding sides.

3 Not enough

ANSWER HA · 3

i. **No.** Triangles with sides 3, 4, 5 cm and 6, 8, 10 cm have the same angles but different sizes.
ii. **Yes.** They are similar, so their sides are in one ratio. Equal perimeters make that ratio 1 : 1, so all the sides are equal: SSS.
iii. **Not always.** If the equal angle lies between the equal sides, SAS makes them congruent. If it doesn't, that is SSA, which doesn't prove congruence.

MARKING SCHEME

- ✓ i. No, with an example.
- ✓ ii. Yes: the ratio must be 1 : 1.
- ✓ iii. Only if the angle lies between the sides (SSA isn't a test).

4 Find the unknowns

ANSWER A · 3

$AB = DE$: $2x + 1 = 3x - 4$, so $x = \mathbf{5}$ and $AB = \mathbf{11\ cm}$.
$BC = EF$: $y + 2 = 2y - 3$, so $y = \mathbf{5}$ and $BC = \mathbf{7\ cm}$.

MARKING SCHEME

- ✓ The equations from the corresponding sides.
- ✓ $x = 5$ and $y = 5$.
- ✓ $AB = 11$ cm and $BC = 7$ cm.

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5 Spot the error

ANSWER U · 3

The letters must be in matching order. $\triangle ABC \cong \triangle PQR$ would say $AB = PQ$, but here $AB = PR$.
B is where AB and BC meet, and the sides equal to them, PR and PQ, meet at P. So B goes with P, A with R and C with Q.
$\triangle ABC \cong \triangle RPQ$, and $\angle B = \angle P$.

MARKING SCHEME

- ✓ The letters aren't in matching order.
- ✓ $\triangle ABC \cong \triangle RPQ$.
- ✓ $\angle B = \angle P$.

6 A line across a triangle

ANSWER HA · 4

i. $\angle A$ is common; $\angle ADE = \angle ABC$ and $\angle AED = \angle ACB$ (corresponding angles, $DE \parallel BC$).

ii. $AD : AB = 4 : 10 = \mathbf{2 : 5}$.

iii. $\frac{DE}{BC} = \frac{2}{5}$, so $BC = 5 \times \frac{5}{2} = \mathbf{12.5\ cm}$.

iv. $\frac{AE}{AC} = \frac{2}{5}$, so $AC = 15$ cm and $EC = 15 - 6 = \mathbf{9\ cm}$.

MARKING SCHEME

- ✓ The equal angles, with reasons.
- ✓ 2 : 5.
- ✓ $BC = 12.5$ cm.
- ✓ $EC = 9$ cm.

7 In a real setting

ANSWER A · 3

i. Both have a right angle at the ground, and the sun's rays come in at the same angle, so all the angles are equal.

ii. $\frac{h}{6} = \frac{1.6}{1.2}$, so $h = 6 \times \frac{4}{3} = \mathbf{8\ m}$.

MARKING SCHEME

- ✓ Equal angles: a right angle and the sun's angle.
- ✓ The ratio of corresponding sides.
- ✓ 8 m.

8 Think

ANSWER HA · 3

i. **Equilateral triangles: yes.** All their angles are 60°. **Squares: yes.** All their angles are 90°, and their sides are in one ratio. **Rectangles: no.** Their angles are equal, but a 2 by 1 and a 3 by 1 rectangle don't have their sides in one ratio.

ii. Areas: $\frac{1}{2} \times 3 \times 4 = 6\ \text{cm}^2$ and $\frac{1}{2} \times 6 \times 8 = 24\ \text{cm}^2$: **4 times**. Both the base and the height have doubled, so the area is 2×2 times as big.

MARKING SCHEME

- ✓ Equilateral triangles and squares are all similar.
- ✓ Rectangles aren't: equal angles aren't enough.
- ✓ 4 times, since the base and the height both double.

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9 Make your own

ANSWER HA · 2

The first perimeter is 15 cm, so the ratio is 45 ÷ 15 = 3, and the new sides are **9, 15 and 21 cm**.
For example, 2, 3, 4 cm and 5, 7.5, 10 cm, with a ratio of 2 : 5.

MARKING SCHEME

- ✓ 9, 15 and 21 cm.
- ✓ Another pair whose sides are in one ratio.

See it at Science Playground

Exhibits and plants at the center, Bhaktapur, that go with this unit. Book a class visit at science-playground.com/booking.

- **Tangram Puzzle** (exhibit): Lay the two largest triangles one on the other: side on side and corner on corner, they cover exactly, so they are congruent. So are the two smallest.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	3	9	15	**27**

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