Unit 13 · Lines and Angles

HARDER PRACTICE QUESTIONS

Full marks 26

Write your answers on separate paper. Show your working, and give the reason beside each step ("alternate angles are equal"). Draw a figure for every question.

1 Find every angle [3]

$AB \parallel CD$, and a transversal cuts them. A pair of co-interior angles are in the ratio 2 : 3.

i. Find the two angles.

ii. The transversal makes eight angles in all. How many different sizes are there, and what do all eight add up to?

2 With algebra [3]

A transversal cuts PQ and RS. A pair of corresponding angles are $(3x - 10)^\circ$ and $(2x + 25)^\circ$.

i. For what value of x are the lines parallel?

ii. If $x = 30$, are the lines parallel? Give a reason.

3 The bent line [3]

$AB \parallel CD$, with B to the right of A and D to the right of C. A point E lies between the lines, to the right of A and C. $\angle BAE = 40^\circ$ and $\angle DCE = 35^\circ$. Find $\angle AEC$.

4 Show it [3]

In $\triangle ABC$, the side BC is produced to D. By drawing $CE \parallel BA$, show that $\angle ACD = \angle BAC + \angle ABC$.

5 Two bisectors [3]

$AB \parallel CD$. A transversal cuts AB at G and CD at H. The bisectors of $\angle BGH$ and $\angle DHG$ meet at K. Find $\angle GKH$. Would your answer change if the transversal leaned over more?

6 Spot the error [3]

Find the mistake and correct it.

A transversal cuts two lines. Basanti measures one angle as 110° and writes:
"Alternate angles are always equal, so the alternate angle is 110°. Co-interior angles are always equal too, so the co-interior angle is 110°. So the lines are parallel."

7 In a real setting [4]

A simple periscope has two parallel mirrors. A ray of light strikes the top mirror at an angle of 45° to it, bounces off at the same angle to the mirror, travels to the bottom mirror, and bounces off that in the same way.

i. What angle does the ray make with the bottom mirror when it arrives? Give a reason.

ii. At each mirror, what is the angle between the ray arriving and the ray leaving?

iii. Show that the ray that leaves is parallel to the ray that came in.

iv. If the mirrors stayed parallel but were tilted so that the first angle was 40°, would the rays still be parallel?

8 Think [2]

Lines a and b are each parallel to line c. Use a transversal across all three to explain why a and b must be parallel to each other.

9 Make your own [2]

Make up expressions in x for a pair of co-interior angles of parallel lines, so that $x = 20$ and the two angles are different. Find the two angles.