# Unit 11 · Algebraic Fraction

**HARDER PRACTICE QUESTIONS · ANSWER KEY**

Full marks 29

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

## 1 No value, and zero

**ANSWER** A · 3

i. $x^2 - 5x + 6 = (x - 2)(x - 3)$, which is 0 when $x =$ **2 or 3**.

ii. The numerator must be 0: $x = \mathbf{-1}$.

iii. $\dfrac{4 + 1}{16 - 20 + 6} = \dfrac{\mathbf{5}}{\mathbf{2}}$.

**MARKING SCHEME**

✓ 2 or 3.

✓ $-1$.

✓ $\dfrac{5}{2}$.

## 2 Two at a time

**ANSWER** HA · 3

$$\frac{1}{x-1} - \frac{1}{x+1} = \frac{(x+1)-(x-1)}{x^2-1} = \frac{2}{x^2-1}.$$

$$\frac{2}{x^2-1} - \frac{2}{x^2+1} = \frac{2(x^2+1)-2(x^2-1)}{(x^2-1)(x^2+1)} = \frac{4}{(x^2-1)(x^2+1)} = \frac{\mathbf{4}}{\mathbf{x^4-1}}.$$

**MARKING SCHEME**

✓ The first two give $\dfrac{2}{x^2-1}$.

✓ The numerator with the third is 4.

✓ $\dfrac{4}{x^4-1}$.

## 3 Signs that turn round

**ANSWER** HA · 4

Rewrite: $\dfrac{1}{(a-b)(a-c)} - \dfrac{1}{(a-b)(b-c)} + \dfrac{1}{(a-c)(b-c)}$.

The LCM of the denominators is $(a-b)(b-c)(a-c)$.

The numerator is $(b-c) - (a-c) + (a-b) = b - c - a + c + a - b = 0$.

So the sum is **0**.

**MARKING SCHEME**

✓ The signs changed so the denominators use the same three factors.

✓ The LCM $(a-b)(b-c)(a-c)$.

✓ The numerator $(b-c) - (a-c) + (a-b)$.

✓ It comes to 0.



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## 4 Brackets first

**ANSWER** A · 3

The bracket: $\frac{x(x+2)-x(x-2)}{x^2-4} = \frac{4x}{x^2-4}$.

Then $\frac{4x}{x^2-4} \times \frac{x^2-4}{4x} = \mathbf{1}$.

**MARKING SCHEME**

- ✓ The bracket is $\frac{4x}{x^2-4}$.
- ✓ Division changed to multiplying by the fraction turned over.
- ✓ 1.

## 5 Spot the error

**ANSWER** U · 3

i. $x^2$ is a **term**, not a factor, so it can't be canceled. The fraction is already in its lowest terms ($x^2 + 5$ has no factors).

ii. The minus sign must go on all of $x + 4$: put a bracket round it. $\frac{5-(x+4)}{x-1} = \frac{1-x}{x-1} = \mathbf{-1}$.

**MARKING SCHEME**

- ✓ i. Terms can't be canceled, only factors.
- ✓ ii. The numerator needs a bracket after the minus sign.
- ✓ ii. The answer is $-1$.

## 6 Simplify, then put the number in

**ANSWER** A · 2

$\frac{(x+2)(x-2)}{(x+1)(x-2)} = \frac{x+2}{x+1}$. When $x = 99$: $\mathbf{\frac{101}{100}}$.

**MARKING SCHEME**

- ✓ $\frac{x+2}{x+1}$.
- ✓ $\frac{101}{100}$.

## 7 In a real setting

**ANSWER** HA · 4

i. $\frac{d}{u} + \frac{d}{v} = \frac{d(u+v)}{uv}$.

ii. $2d \div \frac{d(u+v)}{uv} = 2d \times \frac{uv}{d(u+v)} = \frac{2uv}{u+v}$.

iii. $\frac{2 \times 40 \times 60}{100} = \mathbf{48\ km\ per\ hour}$. The bus spends longer at the slower speed, so the average is nearer 40 than 60.

**MARKING SCHEME**

- ✓ The total time $\frac{d(u+v)}{uv}$.
- ✓ The division, giving $\frac{2uv}{u+v}$.
- ✓ 48 km per hour.
- ✓ More time is spent at the slower speed.

## 8 A sum that collapses

**ANSWER**

i. $\frac{(x+1)-x}{x(x+1)} = \frac{1}{x(x+1)}$.



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ii. Each fraction is a difference: $\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\ldots+\left(\frac{1}{9}-\frac{1}{10}\right)$. Everything cancels except the first and last: $1-\frac{1}{10}=\mathbf{\frac{9}{10}}$. HA · 3

**MARKING SCHEME**

- ✓ i. shown with the LCM $x(x+1)$.
- ✓ ii. Each fraction written as a difference.
- ✓ $\frac{9}{10}$.

## 9 Think

**ANSWER** HA · 2

At $x=3$, both are 5. At $x=2$, $x+2=4$, but the fraction has a denominator of 0, so it **has no value**. So they agree everywhere **except at $x=2$**. Canceling $x-2$ is allowed only where $x-2$ isn't 0.

**MARKING SCHEME**

- ✓ Both are 5 at $x=3$, but the fraction has no value at $x=2$.
- ✓ Canceling needs $x-2\neq 0$, so they aren't exactly the same.

## 10 Make your own

**ANSWER** HA · 2

Sum: $\frac{1}{x-1}+\frac{1}{x+1}=\frac{(x+1)+(x-1)}{x^2-1}=\frac{2x}{x^2-1}$.

Difference: $\frac{x}{x-1}-\frac{x}{x+1}=\frac{x(x+1)-x(x-1)}{x^2-1}=\frac{2x}{x^2-1}$.

**MARKING SCHEME**

- ✓ A pair with the right sum.
- ✓ A pair with the right difference.

| Marks by level | Knowledge | Understanding | Application | Higher ability | Total |
|---|---|---|---|---|---|
| **Marks** | 0 | 3 | 8 | 18 | **29** |



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