Unit 10 · Algebraic Expression

HARDER PRACTICE QUESTIONS · ANSWER KEY

Full marks 29

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 A square in disguise

ANSWER HA · 3

$a^4 + 4 = a^4 + 4a^2 + 4 - 4a^2 = (a^2 + 2)^2 - (2a)^2$

$= \mathbf{(a^2 + 2a + 2)(a^2 - 2a + 2)}$.

(Check, $a = 2$: $16 + 4 = 20$, and $10 \times 2 = 20$. ✓)

MARKING SCHEME

- ✓ $4a^2$ added and subtracted.
- ✓ $(a^2 + 2)^2 - (2a)^2$.
- ✓ $(a^2 + 2a + 2)(a^2 - 2a + 2)$.

2 Without a calculator

ANSWER A · 3

i. $(105 + 95)(105 - 95) = 200 \times 10 = \mathbf{2{,}000}$.

ii. $(1000 - 1)(1000 + 1) = 1000^2 - 1^2 = \mathbf{9{,}99{,}999}$.

iii. $98^2 + 2 \times 98 \times 2 + 2^2 = (98 + 2)^2 = 100^2 = \mathbf{10{,}000}$.

MARKING SCHEME

- ✓ i. 2,000, as a difference of squares.
- ✓ ii. 9,99,999, as a difference of squares.
- ✓ iii. 10,000, as a perfect square.

3 Make it perfect

ANSWER HA · 3

i. $(x + 6)^2 = x^2 + 12x + 36$ and $(x - 6)^2 = x^2 - 12x + 36$, so $k = \mathbf{12}$ **or** $\mathbf{-12}$.

ii. $9x^2 - 24x + k = (3x)^2 - 2 \times 3x \times 4 + k$, so $k = 4^2 = \mathbf{16}$.

MARKING SCHEME

- ✓ $k = 12$.
- ✓ $k = -12$ as well.
- ✓ $k = 16$.

4 A rectangle from its area

ANSWER A · 3

i. Product $6 \times (-20) = -120$, sum 7: the numbers 15 and -8.

$6x^2 + 15x - 8x - 20 = 3x(2x + 5) - 4(2x + 5) = (3x - 4)(2x + 5)$. The sides are $\mathbf{3x - 4}$ **and** $\mathbf{2x + 5}$.

ii. $2(3x - 4 + 2x + 5) = \mathbf{10x + 2}$.

iii. Sides **5 and 11**, area **55**, perimeter **32**. Check: $6 \times 9 + 21 - 20 = 55$. ✓

MARKING SCHEME

- ✓ The sides $3x - 4$ and $2x + 5$.
- ✓ The perimeter $10x + 2$.
- ✓ 5, 11, 55 and 32.

5 The missing expression

ANSWER

$x^2 - 4 = (x - 2)(x + 2)$. The HCF is already in it, and the LCM has one factor it lacks, $x + 3$.

So the other expression must hold the HCF, $x - 2$, and the missing factor, $x + 3$, but not $x + 2$, or the

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HCF would be bigger. HA · 3

It is $(x - 2)(x + 3) = \mathbf{x^2 + x - 6}$.

MARKING SCHEME

✓ $x^2 - 4 = (x - 2)(x + 2)$.

✓ It needs the factors $x - 2$ and $x + 3$, without $x + 2$.

✓ $x^2 + x - 6$.

6 Spot the error

ANSWER U · 3

i. The squares are $(2x)^2$ and 3^2, so $4x^2 - 9 = \mathbf{(2x + 3)(2x - 3)}$.

ii. The product must be -14, so the numbers have opposite signs: -7 and $+2$. $x^2 - 5x - 14 = \mathbf{(x - 7)(x + 2)}$.

Multiplying out his answer: $(x - 7)(x - 2) = x^2 - 9x + 14$, which isn't the expression.

MARKING SCHEME

✓ i. $(2x + 3)(2x - 3)$.

✓ ii. $(x - 7)(x + 2)$.

✓ Multiplying out gives $x^2 - 9x + 14$, which is different.

7 In a real setting

ANSWER A · 4

i. $x^2 + 5x + 6 = (x + 2)(x + 3)$ and $x^2 + 7x + 12 = (x + 3)(x + 4)$. The HCF is $\mathbf{x + 3}$ **m**.

ii. $\mathbf{x + 2}$ pieces and $\mathbf{x + 4}$ pieces.

iii. Ropes **90 m and 110 m**, pieces **10 m**, and $9 + 11 =$ **20 pieces**.

MARKING SCHEME

✓ Both expressions factorized.

✓ The HCF $x + 3$.

✓ $x + 2$ and $x + 4$ pieces.

✓ 90 m, 110 m, 10 m and 20 pieces.

8 Show it

ANSWER HA · 3

Difference of squares: $(2n + 1)^2 - (2n - 1)^2 = [(2n + 1) + (2n - 1)][(2n + 1) - (2n - 1)] = 4n \times 2 = 8n$, a multiple of 8.

Test: $15^2 - 13^2 = 225 - 169 = 56 = 8 \times 7$, with $n = 7$. ✓

MARKING SCHEME

✓ The difference of squares used on the two numbers.

✓ The result is $8n$.

✓ $225 - 169 = 56 = 8 \times 7$.

9 Think

ANSWER HA · 2

$(x + a)(x + b) = x^2 + (a + b)x + ab$, so $a + b = 0$ and $ab = 4$.

$a + b = 0$ means $b = -a$, so $ab = -a^2$, which is never positive. It can't be 4, so $\mathbf{x^2 + 4}$ **doesn't factorize**.

MARKING SCHEME

✓ $a + b = 0$ and $ab = 4$.

✓ $b = -a$ makes ab zero or negative, never 4.

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10 Make your own

HA • 2

ANSWER

The pairs with product 12: 1 and 12, 2 and 6, 3 and 4, and the same pairs negative. So $p = \pm 13, \pm 8$ or ± 7.

$(x + 1)(x + 12)$, $(x + 2)(x + 6)$, $(x + 3)(x + 4)$, and $(x - 1)(x - 12)$, $(x - 2)(x - 6)$, $(x - 3)(x - 4)$.

MARKING SCHEME

- ✓ The three positive values, 13, 8 and 7, with their factors.
- ✓ The three negative values as well.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	3	10	16	**29**

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