Unit 9 · Indices

HARDER PRACTICE QUESTIONS

Full marks 29

Write your answers on separate paper. Show your working. Letters in the indices stand for whole numbers, and bases are not 0.

1 Same base, different look [3]

Write 4^{15}, 8^{10} and 16^{8} as powers of 2. Which two are equal, and which is the largest? How many times as big as the others is it?

2 Find x [3]

Find x:

i. $3^{x} \times 9 = 243$

ii. $4^{x} = 8^{x-1}$

3 Break it into primes [3]

Without a calculator, simplify $\dfrac{6^{4} \times 10^{3}}{15^{2} \times 4^{3}}$.

4 The same for every n [3]

Show that $\dfrac{2^{n+3} - 2^{n+1}}{3 \times 2^{n}}$ has the same value for every whole number n, and find it.

5 Prove it [3]

Prove that $\dfrac{x^{a+b} \times x^{b+c} \times x^{c+a}}{(x^{a} \times x^{b} \times x^{c})^{2}} = 1$.

6 Spot the error [3]

Each line has one mistake. Find it and correct it.

i. $2^{3} \times 2^{4} = 4^{7}$

ii. $(x^{2})^{3} = x^{5}$

iii. $3x^{-2} = \dfrac{1}{3x^{2}}$

7 Negative indices [3]

i. Is $2^{-1} + 3^{-1}$ equal to $(2+3)^{-1}$? Work out both.

ii. Find the value of $(4^{-1} + 2^{-2})^{-1}$.

8 In a real setting [3]

A sheet of paper is 0.1 mm thick. Each fold doubles the thickness of the pile. (Paper can't really be folded more than a few times, so imagine it.)

i. Write the thickness after n folds as a power of 2 times 0.1 mm.

ii. How thick is it after 10 folds, in cm?

iii. Using $2^{10} \approx 10^{3}$, about how many kilometers thick is it after 30 folds?

9 Think [3]

Look at the pattern $2^{3} = 8$, $2^{2} = 4$, $2^{1} = 2$. What is done to get from each line to the next? Carry on the pattern to find 2^{0}, 2^{-1} and 2^{-2}. Why would it be strange if 2^{0} were 0?

10 Make your own [2]

Find four different ways of writing 64 as a whole number or fraction with an index, at least one of them with a negative index.