Unit 9 · Indices

HARDER PRACTICE QUESTIONS · ANSWER KEY

Full marks 29

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Same base, different look

ANSWER A · 3

$4^{15} = (2^2)^{15} = 2^{30}$, $8^{10} = (2^3)^{10} = 2^{30}$, $16^8 = (2^4)^8 = 2^{32}$.

4^{15} and 8^{10} are equal, and **16^8 is the largest**.

$2^{32} \div 2^{30} = 2^2 =$ **4 times** as big.

MARKING SCHEME

- ✓ All three as powers of 2.
- ✓ $4^{15} = 8^{10}$, and 16^8 is the largest.
- ✓ 4 times.

2 Find x

ANSWER HA · 3

i. $3^x \times 3^2 = 3^5$, so $x + 2 = 5$ and $x = \mathbf{3}$.

ii. $(2^2)^x = (2^3)^{x-1}$, so $2^{2x} = 2^{3x-3}$. Then $2x = 3x - 3$ and $x = \mathbf{3}$.

(Check: $4^3 = 64 = 8^2$. ✓)

MARKING SCHEME

- ✓ i. $x = 3$.
- ✓ ii. Both sides as powers of 2.
- ✓ ii. $x = 3$.

3 Break it into primes

ANSWER A · 3

$$\frac{2^4 \times 3^4 \times 2^3 \times 5^3}{3^2 \times 5^2 \times 2^6} = 2^{4+3-6} \times 3^{4-2} \times 5^{3-2} = 2 \times 9 \times 5 = \mathbf{90}.$$

MARKING SCHEME

- ✓ Every base written in prime factors.
- ✓ $2^1 \times 3^2 \times 5^1$.
- ✓ 90.

4 The same for every n

ANSWER HA · 3

$2^{n+3} = 2^n \times 8$ and $2^{n+1} = 2^n \times 2$, so the top is $2^n \times (8 - 2) = 2^n \times 6$.

$\frac{2^n \times 6}{3 \times 2^n} =$ **2**, whatever n is.

(Check, $n = 3$: $\frac{64 - 16}{24} = 2$. ✓)

MARKING SCHEME

- ✓ $2^{n+3} = 2^n \times 2^3$ and $2^{n+1} = 2^n \times 2$.
- ✓ 2^n taken out and canceled.
- ✓ 2.

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5 Prove it

ANSWER A · 3

Top: $x^{(a+b)+(b+c)+(c+a)} = x^{2a+2b+2c}$.

Bottom: $(x^{a+b+c})^2 = x^{2a+2b+2c}$.

Top ÷ bottom $= x^0 = 1$.

MARKING SCHEME

- ✓ The top is $x^{2a+2b+2c}$.
- ✓ The bottom is $x^{2a+2b+2c}$.
- ✓ $x^0 = 1$.

6 Spot the error

ANSWER U · 3

i. The base stays the same; only the indices are added: $2^3 \times 2^4 = \mathbf{2^7}$.

ii. A power of a power multiplies the indices: $(x^2)^3 = \mathbf{x^6}$.

iii. The index -2 belongs to x only, so the 3 stays on top: $3x^{-2} = \dfrac{\mathbf{3}}{\mathbf{x^2}}$.

MARKING SCHEME

- ✓ i. 2^7.
- ✓ ii. x^6.
- ✓ iii. $\dfrac{3}{x^2}$.

7 Negative indices

ANSWER A · 3

i. $2^{-1} + 3^{-1} = \dfrac{1}{2} + \dfrac{1}{3} = \dfrac{5}{6}$, but $(2+3)^{-1} = \dfrac{1}{5}$. **They aren't equal.**

ii. $\dfrac{1}{4} + \dfrac{1}{4} = \dfrac{1}{2}$, and $\left(\dfrac{1}{2}\right)^{-1} = \mathbf{2}$.

MARKING SCHEME

- ✓ $\dfrac{5}{6}$ and $\dfrac{1}{5}$, so not equal.
- ✓ The bracket is $\dfrac{1}{2}$.
- ✓ 2.

8 In a real setting

ANSWER A · 3

i. $\mathbf{2^n \times 0.1\ mm}$.

ii. $2^{10} \times 0.1 = 102.4$ mm $=$ **10.24 cm**.

iii. $2^{30} = (2^{10})^3 \approx 10^9$, so $10^9 \times 0.1 = 10^8$ mm. 10^8 mm $= 10^5$ m $=$ **100 km**.

MARKING SCHEME

- ✓ $2^n \times 0.1$ mm.
- ✓ 10.24 cm.
- ✓ About 100 km.

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9 Think

ANSWER HA · 3

Each time the index goes down by 1, the value is **divided by 2**.

So $2^0 = 2 \div 2 = \mathbf{1}$, $2^{-1} = \mathbf{\frac{1}{2}}$ and $2^{-2} = \mathbf{\frac{1}{4}}$.

If 2^0 were 0, halving 2 would give 0 and the pattern would break. And $2^3 \times 2^0$ should be 2^3, not 0.

MARKING SCHEME

- ✓ Each step divides by 2.
- ✓ 1, $\frac{1}{2}$ and $\frac{1}{4}$.
- ✓ 0 would break the pattern (or the law $x^m \times x^0 = x^m$).

10 Make your own

ANSWER HA · 2

For example, 2^6, 4^3, 8^2 and $\left(\frac{1}{2}\right)^{-6}$ (or $\left(\frac{1}{8}\right)^{-2}$, $\left(\frac{1}{4}\right)^{-3}$).

MARKING SCHEME

- ✓ Three ways, each equal to 64.
- ✓ A fourth with a negative index.

See it at Science Playground

Exhibits and plants at the center, Bhaktapur, that go with this unit. Book a class visit at science-playground.com/booking.

- **Growing Dominoes** (exhibit): Each domino is about 1.5 times the one before, so m steps and then n more make it $1.5^m \times 1.5^n = 1.5^{m+n}$ times the first.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	3	15	11	**29**

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