Unit 3 · Rational and Irrational Numbers

HARDER PRACTICE QUESTIONS · ANSWER KEY

Full marks 28

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Rational or irrational?

ANSWER A · 4

Rational: $\sqrt{0.16} = 0.4$; $\sqrt[3]{27} = 3$; $0.\overline{10}$, a recurring decimal; $\frac{22}{7}$, a fraction; $\sqrt{\frac{4}{9}} = \frac{2}{3}$.

Irrational: $\sqrt{0.4} = 0.6324\ldots$, since 0.4 is not the square of a decimal; $0.1010010001\ldots$, which never ends and never repeats (it has a pattern, but no block that repeats); π, since $\frac{22}{7}$ is only close to it.

MARKING SCHEME

- ✓ $\sqrt{0.16}$, $\sqrt[3]{27}$ and $\sqrt{\frac{4}{9}}$ rational, each worked out.
- ✓ $\sqrt{0.4}$ irrational, not the same as $\sqrt{0.16}$.
- ✓ $0.1010010001\ldots$ irrational: no repeating block; $0.\overline{10}$ rational.
- ✓ $\frac{22}{7}$ rational and π irrational.

2 A decimal that starts late

ANSWER HA · 3

Let $x = 0.3181818\ldots$

Then $10x = 3.181818\ldots$ and $1000x = 318.181818\ldots$

Subtracting: $990x = 315$, so $x = \frac{315}{990} = \frac{\mathbf{7}}{\mathbf{22}}$.

(Check: $7 \div 22 = 0.31818\ldots$)

MARKING SCHEME

- ✓ $10x$ and $1000x$, with the same digits after the point.
- ✓ $990x = 315$.
- ✓ $\frac{7}{22}$.

3 On the number line

ANSWER A · 3

i. A triangle with its two shorter sides 3 and 2 units, since $3^2 + 2^2 = 13$, so its hypotenuse is $\sqrt{13}$. Draw it with the side 3 along the line from 0, and the side 2 upward at 3.

ii. Swing the hypotenuse down onto the line with a compass centered at 0. Since $3^2 = 9 < 13 < 16 = 4^2$, the point falls **between 3 and 4**.

iii. With the same compass opening, swing the arc to the left of 0.

MARKING SCHEME

- ✓ Sides 3 and 2, because $9 + 4 = 13$.
- ✓ The arc from 0, landing between 3 and 4.
- ✓ $-\sqrt{13}$ at the same distance to the left of 0.

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking

4 Spot the error

ANSWER U · 3

i. The screen has no room for more digits, so the calculator has rounded. The square of 1.414213562 ends in 4 (its last digit is 2×2), so it can't be exactly 2. $\sqrt{2}$ is **irrational**.

ii. She added the powers, which is what we do when we **multiply**. To add, make the powers the same first: $3.2 \times 10^5 + 0.41 \times 10^5 = \mathbf{3.61 \times 10^5}$.

MARKING SCHEME

- ✓ i. The calculator shows a rounded value; $\sqrt{2}$ is irrational.
- ✓ ii. The powers must be made the same; they aren't added.
- ✓ ii. 3.61×10^5.

5 Put them in order

ANSWER A · 3

As decimals: $4.5 \times 10^{-3} = 0.0045$, $470 \times 10^{-5} = 0.0047$, $0.44 \times 10^{-2} = 0.0044$.
Order: $\mathbf{0.44 \times 10^{-2}}$, $\mathbf{4.5 \times 10^{-3}}$, $\mathbf{470 \times 10^{-5}}$, **0.046**.
Largest: $0.046 = \mathbf{4.6 \times 10^{-2}}$.

MARKING SCHEME

- ✓ The numbers written in one form.
- ✓ The order.
- ✓ 4.6×10^{-2}.

6 Simplify

ANSWER A · 3

Top: $4.8 \times 2.5 = 12$, and $10^5 \times 10^{-3} = 10^2$, so 12×10^2.
Bottom: $16 \times 10^3 - 4 \times 10^3 = 12 \times 10^3$.
$\frac{12 \times 10^2}{12 \times 10^3} = 10^{-1} = \mathbf{1 \times 10^{-1}}$.

MARKING SCHEME

- ✓ The top is 12×10^2.
- ✓ The bottom is 12×10^3, with the powers made the same first.
- ✓ 1×10^{-1}.

7 In a real setting

ANSWER A · 4

i. $\frac{30}{2 \times 10^{-5}} = 15 \times 10^5 = \mathbf{1.5 \times 10^6}$ **grains**.

ii. $(1.5 \times 10^6) \times (6 \times 10^{-3}) = 9 \times 10^3$ m $= \mathbf{9}$ **km**.

MARKING SCHEME

- ✓ The division: $30 \div (2 \times 10^{-5})$.
- ✓ 1.5×10^6 grains.
- ✓ 9×10^3 m.
- ✓ 9 km.

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking

8 Think

ANSWER HA · 3

Let $x = 0.999\ldots$. Then $10x = 9.999\ldots$, and subtracting, $9x = 9$, so $x = 1$. So $0.\overline{9}$ **is exactly 1**.

Another way: $\frac{1}{3} = 0.333\ldots$. Multiply both sides by 3: $1 = 0.999\ldots$.

(No number lies between them, so they can't be two different numbers.)

MARKING SCHEME

- ✓ $10x - x = 9$, so $9x = 9$.
- ✓ $x = 1$: the two are the same number.
- ✓ $3 \times \frac{1}{3} = 3 \times 0.333\ldots$ gives the same.

9 Make your own

ANSWER HA · 2

i. For example, $\sqrt{2}$ and $-\sqrt{2}$: the sum is 0.

ii. The same pair: $\sqrt{2} \times (-\sqrt{2}) = -2$. (Or $3 + \sqrt{2}$ and $3 - \sqrt{2}$, whose sum is 6 and product is 7.)

MARKING SCHEME

- ✓ A pair that has a rational sum.
- ✓ A pair that has a rational product.

See it at Science Playground

Exhibits and plants at the center, Bhaktapur, that go with this unit. Book a class visit at science-playground.com/booking.

- **Patterns from Straight Lines** (exhibit): Find the Spiral of Theodorus on the board: its right triangles have hypotenuses $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and more, irrational lengths you can draw though their digits never end.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	3	17	8	**28**

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking