Unit 2 · Whole Numbers

HARDER PRACTICE QUESTIONS · ANSWER KEY

Full marks 28

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 The missing digits

ANSWER A · 3

i. $2 \times 25 + x \times 5 + 3 = 73$, so $5x = 20$ and $x = \mathbf{4}$.

ii. $8 + 4a + b = 13$, so $4a + b = 5$. With digits 0 or 1 only, $a = \mathbf{1}$ and $b = \mathbf{1}$.

iii. **No.** $1a0b_2$ can only be $1000_2 = 8$, $1001_2 = 9$, $1100_2 = 12$ or $1101_2 = 13$. (11 is 1011_2, which has a 1 in the place that must be 0.)

MARKING SCHEME

- ✓ $x = 4$.
- ✓ $a = 1$, $b = 1$.
- ✓ No, it can only be 8, 9, 12 or 13.

2 Put them in order

ANSWER A · 3

$101101_2 = 32 + 8 + 4 + 1 = 45$. $143_5 = 25 + 20 + 3 = 48$.

Order: $\mathbf{101101_2}$, **46**, $\mathbf{143_5}$.

Difference: $48 - 45 = 3$, which is $\mathbf{11_2}$ in binary and $\mathbf{3_5}$ in quinary.

MARKING SCHEME

- ✓ 45 and 48.
- ✓ The order.
- ✓ 11_2 and 3_5.

3 One string of digits, two numbers

ANSWER HA · 3

i. $1011_5 = 125 + 5 + 1 = 131$ and $1011_2 = 8 + 2 + 1 = 11$. Difference: $131 - 11 = 120$.
$120 = 4 \times 25 + 4 \times 5 + 0$, which is $\mathbf{440_5}$.

ii. Each 1 stands in the same place in both readings, and each place value in quinary (1, 5, 25, …) is at least as big as the same place in binary (1, 2, 4, …). The two are equal only for the digits 0 and 1 alone.

MARKING SCHEME

- ✓ 131 and 11.
- ✓ 120 is 440_5.
- ✓ The place values of quinary are bigger, place by place.

4 Spot the error

ANSWER U · 3

$011001_2 = 16 + 8 + 1 = 25$, not 38.

He read the remainders **from the top down**. They are read from the bottom up: the last remainder is the leftmost digit.

$38 = \mathbf{100110_2}$. (Check: $32 + 4 + 2 = 38$.)

MARKING SCHEME

- ✓ He got back 25.
- ✓ The mistake: the remainders must be read from the bottom up.
- ✓ 100110_2.

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5 Show it

ANSWER HA · 3

i. $1011_2 = 11$ and $10110_2 = 22$: the 0 **doubles** the number. $23_5 = 13$ and $230_5 = 65$: the 0 **multiplies it by 5**.

ii. The 0 moves every digit one place to the left. Each place is 2 times the place on its right in binary, and 5 times in quinary, so every digit is worth 2 or 5 times as much. (It is the same as writing a 0 after a decimal number, which multiplies it by 10.)

MARKING SCHEME

- ✓ 11, 22 and 13, 65.
- ✓ It doubles a binary number and multiplies a quinary number by 5.
- ✓ Every digit moves one place left, and each place is the base times the one on its right.

6 How many digits?

ANSWER A · 4

i. Largest: $4444_5 = 500 + 100 + 20 + 4 =$ **624**. Smallest: $1000_5 =$ **125**.

ii. $624 - 125 + 1 =$ **500**.

iii. $1000 = 1111101000_2$, **10 digits**, and $1000 = 13000_5$, **5 digits**.

iv. **Binary**: its places grow only 2 times at each step, so it takes more of them to reach the same number.

MARKING SCHEME

- ✓ 624 and 125.
- ✓ 500.
- ✓ 10 digits and 5 digits.
- ✓ Binary, with the reason.

7 In a real setting

ANSWER A · 4

i. $389 = 3 \times 125 + 0 \times 25 + 2 \times 5 + 4$, so $389 =$ **3024_5**.

ii. Three 125-rupee notes, no 25-rupee note, two 5-rupee notes and four 1-rupee notes.

iii. $3 + 0 + 2 + 4 =$ **9 notes**.

iv. Five notes of one kind make one note of the next kind, so one note would do the work of five. That is why no quinary digit is more than 4.

MARKING SCHEME

- ✓ 3024_5.
- ✓ Three 125s, two 5s and four 1s.
- ✓ 9 notes.
- ✓ Five notes of one kind make one note of the next.

8 Think

ANSWER HA · 3

$1234_5 = 125 + 50 + 15 + 4 = 194$, which is even, and its digits add up to $1 + 2 + 3 + 4 = 10$, also even.
✓

All the place values 1, 5, 25, 125, … are odd. An odd place value times a digit is odd when the digit is odd and even when the digit is even. So the number has the same oddness as the sum of its digits.

MARKING SCHEME

- ✓ 194, and the digit sum 10: both even.
- ✓ All the place values of quinary are odd.
- ✓ So each digit counts as odd or even just as it stands, and the total behaves like the digit sum.

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9 Make your own

HA · 2

ANSWER

For example, CAT is 00011 00001 10100 (C = 3, A = 1, T = 20).

Five binary digits reach $11111_2 = 31$, which is more than 26. Four reach only $1111_2 = 15$, so P to Z would have no code.

MARKING SCHEME

- ✓ A word written correctly in the code.
- ✓ Five digits reach 31, which covers 26; four reach only 15.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	3	14	11	**28**

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