Unit 1 · Set

HARDER PRACTICE QUESTIONS

Full marks 32

Write your answers on separate paper. Show your working. Write every set by listing unless the question asks for something else.

1 Work backwards [3]

A set M has 127 proper subsets.

i. How many elements has M?
ii. How many subsets has it altogether?
iii. One new element is put into M. How many more subsets does it have now?

2 Subsets with a condition [4]

$P = \{a, b, c, d, e\}$. Without listing them all, find how many subsets of P:

i. contain a
ii. don't contain a
iii. contain neither a nor b
iv. contain a but not b

3 What they share [3]

$M = \{2, 3, 5, 7, 11\}$ and $N = \{1, 3, 5, 7, 9\}$.

i. Which elements do M and N have in common?
ii. How many subsets do M and N have in common? Explain how you know without listing them.
iii. Which is the largest set that is a subset of both? Is it a proper subset of M?

4 The smallest change [3]

$U = \{\text{natural numbers up to } 30\}$, $A = \{\text{multiples of } 4\}$, $B = \{\text{multiples of } 6\}$ and $C = \{\text{odd numbers}\}$, all in U.

i. Which of the pairs A and B, A and C, B and C are disjoint?
ii. What is the least number of elements to take out of B so that A and B become disjoint? Which elements?
iii. One number of U is added to C, and C now overlaps both A and B. Which numbers could it be?

5 Is it possible? [4]

Give a reason for each.

i. A set has exactly 20 subsets.
ii. Two disjoint sets have a subset in common.
iii. A set has no proper subset at all.
iv. A is a proper subset of B, and B is a proper subset of A.

6 Spot the error [3]

Find the mistake and correct it.

> Pratik wrote: "The proper subsets of {2, 4, 6} are {2}, {4}, {6}, {2, 4}, {2, 6} and {4, 6}. So there are 6 proper subsets and, with the improper subset, 7 subsets. The formula for the number of subsets must be $2^n + 1$."

7 In a real setting [4]

A tea stall offers five things to add to a cup of tea: milk, ginger, cardamom, black pepper and lemon. A customer may ask for any of them, or none. (The stall is made up for the problem.)

i. How many different cups can the stall make?
ii. The owner won't put lemon and milk in the same cup. How many cups are left?
iii. How many things would the stall need to offer, with no rule like that, to make at least 100 different cups?

8 Find the sets from the clues [3]

$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$. A has 3 elements and B has 4. Their only common element is a square number greater than 1. All the elements of A are even, and 8 isn't in A. The only even element of B is the common one, and 1 isn't in B.

List A and B, and show them in a Venn diagram with every element of U in its place.

9 Think [3]

Sampada says: "If A is a proper subset of B, and B and C are disjoint, then A and C must be disjoint." Nishant says: "And if B and C overlap, then A and C must overlap."

Who is right? Explain, with an example for anyone who is wrong.

10 Make your own [2]

Make up sets A, B and C inside $U = \{1, 2, 3, \ldots, 9\}$ so that A is a proper subset of B, B and C overlap, A and C are disjoint, and C has exactly 3 elements. Show them in one Venn diagram.