Unit 1 · Set

HARDER PRACTICE QUESTIONS · ANSWER KEY

Full marks 32

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Work backwards

ANSWER A · 3

i. Proper subsets $= 2^n - 1 = 127$, so $2^n = 128 = 2^7$ and $n = \mathbf{7}$.
ii. $2^7 = \mathbf{128}$ subsets (the 127 proper ones and 1 improper).
iii. With 8 elements there are $2^8 = 256$ subsets, so $256 - 128 = \mathbf{128\ more}$. The number of subsets doubles.

MARKING SCHEME

✓ $2^n - 1 = 127$, so $n = 7$.
✓ 128 subsets.
✓ $256 - 128 = 128$ more.

2 Subsets with a condition

ANSWER HA · 4

i. Put a in, then choose any subset of $\{b, c, d, e\}$: $2^4 = \mathbf{16}$.
ii. Any subset of $\{b, c, d, e\}$: $2^4 = \mathbf{16}$. (Check: $16 + 16 = 32 = 2^5$, all the subsets of P.)
iii. Any subset of $\{c, d, e\}$: $2^3 = \mathbf{8}$.
iv. Put a in, leave b out, then choose any subset of $\{c, d, e\}$: $2^3 = \mathbf{8}$.

MARKING SCHEME

✓ i. 16, a free choice from the other four elements.
✓ ii. 16.
✓ iii. 8, a free choice from c, d, e.
✓ iv. 8.

3 What they share

ANSWER HA · 3

i. **3, 5 and 7.**
ii. A set is a subset of both exactly when all its elements are among 3, 5 and 7. So the common subsets are the subsets of $\{3, 5, 7\}$: $2^3 = \mathbf{8}$.
iii. $\{3, 5, 7\}$. **Yes**: every element of it is in M, and M has more elements.

MARKING SCHEME

✓ 3, 5 and 7.
✓ The common subsets are the subsets of $\{3, 5, 7\}$, so 8.
✓ $\{3, 5, 7\}$, a proper subset of M.

4 The smallest change

ANSWER A · 3

$A = \{4, 8, 12, 16, 20, 24, 28\}$, $B = \{6, 12, 18, 24, 30\}$. C has only odd numbers, and A and B have only even ones.

i. A and C, and B and C, are **disjoint**. A and B overlap.
ii. **2 elements**: 12 and 24, the only ones they share.
iii. It must be in both A and B: **12 or 24**.

MARKING SCHEME

✓ A and C, and B and C, are disjoint.
✓ Take out 12 and 24.
✓ 12 or 24.

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5 Is it possible?

ANSWER HA · 4

i. **No.** The number of subsets is 2^n: 1, 2, 4, 8, 16, 32, … and 20 isn't one of them.
ii. **Yes.** The empty set { } is a subset of every set, so it is always common. It is the only one they share.
iii. **Yes**, one set: the empty set. It has $2^0 = 1$ subset, itself, which is improper, and $2^0 - 1 = 0$ proper subsets.
iv. **No.** B would have more elements than A, and A more than B. Both can't be true.

MARKING SCHEME

- ✓ i. No, since 20 is not a power of 2.
- ✓ ii. Yes, the empty set.
- ✓ iii. Yes, the empty set has none.
- ✓ iv. No, each would need more elements than the other.

6 Spot the error

ANSWER U · 3

He has left out the **empty set**, { }, which is a subset of every set, and a proper subset of this one.
So there are **7 proper** subsets and **8 subsets** altogether, and the formula stands: $2^3 = 8$, not $2^n + 1$.

MARKING SCHEME

- ✓ The mistake: the empty set is left out.
- ✓ 7 proper subsets, 8 subsets in all.
- ✓ The formula is 2^n.

7 In a real setting

ANSWER A · 4

i. A cup is a subset of the five things: $2^5 = \mathbf{32}$.
ii. Cups with both lemon and milk: put those two in, then choose any subset of the other three, $2^3 = 8$. Left: $32 - 8 = \mathbf{24}$.
iii. $2^6 = 64$ is too few, and $2^7 = 128$ is enough: **7 things**.

MARKING SCHEME

- ✓ 32, as subsets of a set of 5.
- ✓ 8 cups have both.
- ✓ 24 left.
- ✓ 7 things, since $2^6 = 64 < 100 \leq 128 = 2^7$.

8 Find the sets from the clues

ANSWER A · 3

The square numbers in U greater than 1: only 4. So 4 is common.
A takes three of 2, 4, 6, 8 without 8: $A = \{2, 4, 6\}$.
B is 4 with three odd numbers, not 1: $B = \{3, 4, 5, 7\}$.
In the diagram: 2, 6 in A only; 4 in the overlap; 3, 5, 7 in B only; 1 and 8 outside both circles, inside the rectangle.

MARKING SCHEME

- ✓ 4 is the common element.
- ✓ $A = \{2, 4, 6\}$ and $B = \{3, 4, 5, 7\}$.
- ✓ The diagram, with 1 and 8 outside the circles.

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9 Think

HA · 3

ANSWER

Sampada is right. Every element of A is in B, and no element of B is in C, so no element of A is in C.
Nishant is wrong. The element they share may be one of those in B but not in A. For example, $A = \{1\}$, $B = \{1, 2\}$, $C = \{2, 3\}$: B and C overlap, but A and C are disjoint.

MARKING SCHEME

- ✓ Sampada is right.
- ✓ Her reason: every element of A is in B, and none of B is in C.
- ✓ Nishant is wrong, with an example that works.

10 Make your own

HA · 2

ANSWER

For example, $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{4, 5, 6\}$. In the diagram, A's circle sits inside B's, and C's circle crosses B's but not A's. Any sets that fit all four conditions are right.

MARKING SCHEME

- ✓ Three sets that fit all four conditions.
- ✓ One Venn diagram that shows them correctly.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	3	13	16	**32**

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