Unit 19 · Transformation

STUDY NOTES

WHAT THE BLE ASKS

- **CDC's grid:** Geometry (Units 13, 14, 15, 16, 17, 18, 19, 20 together): 15 of 50 marks
- **In our 10 model papers:** 2-mark ×1 · 3-mark ×9
- **Model paper, question 11 (3 marks, application):** a triangle on a graph: rotate it through +90° about the origin and write the coordinates of the image
- the Grade 7 model paper shows two more shapes: reflect points in the X-axis (2 marks); name a transformation with a reason (1 mark)

Key terms

Transformation: moves a point or a shape by a fixed rule.

Object: the shape you start with.

Image: where it ends up; the image of A is A', read "A-dash".

Reflection: flips a shape over a line, the **mirror line**.

Translation: shifts a shape in a given direction through a given distance.

Rotation: turns a shape about a fixed point.

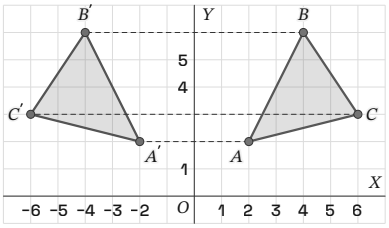
Rules

$(x, y) \to (x, -y)$	reflection in the X-axis **Use:** $(3, 5) \to (3, -5)$
$(x, y) \to (-x, y)$	reflection in the Y-axis **Use:** $(3, 5) \to (-3, 5)$
$(x, y) \to (x + a, y + b)$	translation by (a, b) **Use:** right and up +; left and down −
$(x, y) \to (-y, x)$	+90° about O, anticlockwise **Use:** $(4, 6) \to (-6, 4)$
$(x, y) \to (y, -x)$	−90° about O, clockwise **Use:** $(4, 6) \to (6, -4)$
$(x, y) \to (-x, -y)$	180° about O **Use:** $(4, 6) \to (-4, -6)$

In one line: a flip changes one sign, a slide adds, and a quarter turn swaps the two numbers and changes one sign.

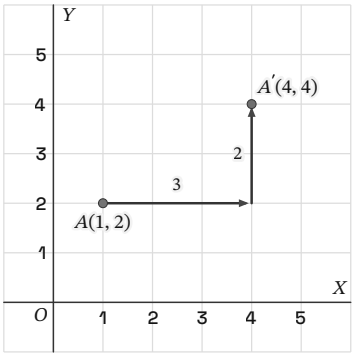
Reflection

- each point and its image: the same distance from the mirror, on opposite sides
- the line joining them crosses the mirror at a right angle
- a point on the mirror doesn't move: in the Y-axis, $(0, 4) \to (0, 4)$
- the number that changes is **not** the one the axis names: the X-axis runs across, so the point moves up or down, and y changes
- image congruent to the object, but **turned over**: $A \to B \to C$ anticlockwise becomes $A' \to B' \to C'$ clockwise
- $P(5, -6)$ in the Y-axis: $P'(-5, -6)$; $PP' = 5 - (-5) = 10$ units



$A(2, 2)$, $B(4, 6)$, $C(6, 3)$ in the Y-axis: $A'(-2, 2)$, $B'(-4, 6)$, $C'(-6, 3)$.

Translation



Three units right, then two units up: the rule $(3, 2)$.

- every point moves the same distance in the same direction

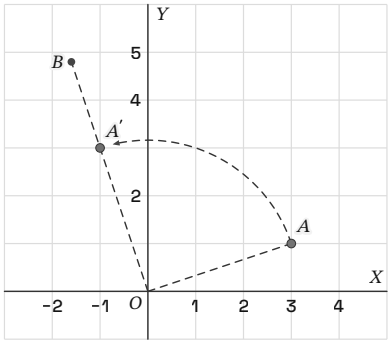
- "4 left and 3 down" is the rule $(-4, -3)$
- the rule from a point and its image: subtract: $(-3, 5) \to (4, 5)$: $(4 - (-3), 5 - 5) = (7, 0)$, 7 right
- right then up, or up then right: the same point
- going left always makes x smaller: 3 left of -2 is $-2 - 3 = -5$
- image congruent to the object, and not turned over

Rotation

Three things are needed:

1. the **center of rotation**: the point that stays still
2. the **angle of rotation**: how far it turns (45°, 90° or 180°)
3. the **direction of rotation**: which way it turns

- anticlockwise is **positive**; clockwise, the way of a clock's hands, is **negative**
- +90° is a quarter turn anticlockwise; −90° a quarter turn clockwise
- 180° is the same either way, so it needs no sign; it is two quarter turns
- each point stays the same distance from the center, along an arc
- image congruent to the object, and not turned over



$OA' = OA$, and the angle AOA' is 90°.

With a protractor:

1. join the point to the center
2. measure the angle from that line, in the right direction
3. mark the image as far from the center as the point is

Why the numbers swap: to reach (x, y), go x across and y up. After +90° the "across" goes up and the "up" goes left: y left and x up, $(-y, x)$.

Test your rule: $(1, 0)$ is to the right of O; a quarter turn anticlockwise must take it to the top, $(0, 1)$. The rule $(-y, x)$ gives $(0, 1)$.

A shape on a graph

1. write the coordinates of each vertex
2. use the rule on each vertex
3. plot the images and join them
4. write the image's coordinates

Two kinds of move in a row: follow them in order. Flip in the Y-axis, then slide 2 right: $(1, 1) \to (-1, 1) \to (1, 1)$. Slide first: $(1, 1) \to (3, 1) \to (-3, 1)$.

Where it is used

- games: one drawing of a sprite is flipped to face left; pygame's `rotate` uses the same signs as this unit
- phones: every scroll is a translation; rotating a photo is a quarter turn
- ambulances: the word is painted mirrored; a driver's mirror undoes it

COMMON MISTAKES

✗ reflecting in the wrong axis
✓ the X-axis changes y.

✗ turning the wrong way
✓ positive is anticlockwise.

✗ swapping the numbers, forgetting the sign
✓ a quarter turn does both.

✗ losing a sign on a negative coordinate
✓ $-(-5) = 5$.

✗ writing the images but not drawing them
✓ show them when the question says so.