Unit 18 · Tessellation

STUDY NOTES

WHAT THE BLE ASKS

- **CDC's grid:** Geometry (Units 13, 14, 15, 16, 17, 18, 19, 20 together): 15 of 50 marks
- **In our 10 model papers:** 1-mark ×6 · 2-mark ×4
- **Model paper, Geometry part a (1 mark, knowledge):** "Which type of triangle is used to make regular tessellation?": equilateral
- the Grade 6 paper asks you to make one and color it (2 marks, higher ability), so a short drawing part can come too

Key terms

Tessellation: a covering of a flat surface by shapes, with **no gaps and no overlaps**.

Tiles: the shapes.

Regular polygon: all sides equal and all angles equal.

Regular tessellation: one kind of regular polygon, all the same size, whole sides meeting whole sides.

Semi-regular tessellation: two or more kinds of regular polygon, whole sides meeting whole sides, **the same arrangement at every corner**.

Formulas

$\frac{(n-2)\times 180^\circ}{n}$	n sides **Use:** one angle of a regular polygon
$n = 360^\circ \div$ exterior angle	exterior angle $= 180^\circ -$ angle **Use:** sides from an angle
360°	round every corner **Use:** the angles at a corner

Naming a tessellation

1. pick any corner where the tiles meet
2. start at a tile with the fewest sides
3. go round the corner **in order**; write each tile's number of sides, with dots between

- one number for every tile at the corner: squares are 4.4.4.4, never 4.4.4
- "from small to large" only says where to start: 3.3.4.3.4, 3.6.3.6 aren't in size order
- 3.3.3.4.4 and 3.3.4.3.4 are two different floors

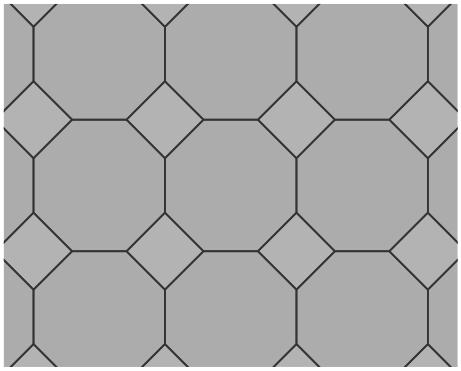
Regular tessellations: only three

Regular polygon	One angle	360° ÷ angle	A floor?
Triangle	60°	6	Yes: 3.3.3.3.3.3
Square	90°	4	Yes: 4.4.4.4
Pentagon	108°	3.33…	No
Hexagon	120°	3	Yes: 6.6.6

- the angle must go into 360° exactly
- at least three tiles at a corner, so an angle can't be over $360^\circ \div 3 = 120^\circ$
- bricks: a tessellation, but not regular (a brick isn't a regular polygon)
- squares set like bricks: not regular either: corners land in the middle of a side

Semi-regular tessellations

- round each corner the angles add up to 360°; they needn't be equal
- for the exam: "two or more regular polygons, with the same arrangement at every corner"
- adding up to 360° is **needed but not enough**: 3.3.6.6 closes round a point, but can't be every corner of a floor



4.8.8: regular octagons alone leave gaps; a small square fits each gap exactly (135° + 135° + 90°).

Missing tile at a corner:

1. take the angles you know from 360°
2. what is left is the tile's angle
3. exterior angle = 180° − angle
4. $n = 360° \div$ exterior angle

- a square and two equal tiles: $(360° - 90°) \div 2 = 135°$; exterior 45°; $360 \div 45 = 8$: octagon, 4.8.8
- triangle, two squares: $360° - 240° = 120°$: hexagon, 3.4.6.4

Name	Tiles	Angles at a corner
3.3.3.3.6	triangles, hexagons	60° × 4, 120°
3.3.3.4.4	triangles, squares	60° × 3, 90° × 2
3.3.4.3.4	triangles, squares	60° × 3, 90° × 2
3.4.6.4	triangles, squares, hexagons	60°, 90° × 2, 120°
3.6.3.6	triangles, hexagons	60° × 2, 120° × 2
3.12.12	triangles, 12-gons	60°, 150° × 2
4.6.12	squares, hexagons, 12-gons	90°, 120°, 150°
4.8.8	squares, octagons	90°, 135° × 2

- exactly **eleven** floors: three regular, eight semi-regular
- twenty-one arrangements close round a point; these eleven spread

Tiles that aren't regular

- **any triangle:** turn a copy half way round the middle of a side → a parallelogram; parallelograms fit in rows; $2 \times 180° = 360°$ at a corner
- **any four-sided shape:** the four angles add up to 360°, one of each round every corner
- **from a square:** cut a piece from one side, stick it on the opposite side; same area; the bump fits the next dent
- circles alone: gaps; a tessellation is about the whole set of tiles

Where it is used

- bricks, paving; herringbone interlocks the most
- James Webb telescope's mirror: 18 hexagons, 6.5 m across
- graphene: carbon atoms in a 6.6.6 pattern
- football: pentagon + two hexagons: 108° + 120° + 120° = 348°, 12° short: the surface curves

COMMON MISTAKES

✗ not knowing what a regular polygon is
✓ all sides equal and all angles equal.

✗ angles round a corner not making 360°
✓ they must.

✗ numbers in size order, 3.3.6.6 for 3.6.3.6
✓ go round the corner.

✗ the wrong number of numbers: 4.4.4 for squares
✓ one for every tile: 4.4.4.4.

✗ any mix called semi-regular
✓ check every corner is alike.