# Unit 15 · Congruency and Similarity

STUDY NOTES

**WHAT THE BLE ASKS**

- **CDC's grid:** Geometry (Units 13, 14, 15, 16, 17, 18, 19, 20 together): 15 of 50 marks
- **In our 10 model papers:** 2-mark ×10
- **Model paper, Geometry part b (2 marks, higher ability):** cut the parallelogram along a diagonal; prove the two triangles congruent
- 1 mark: the three pairs with their reasons
- 1 mark: the test named, the letters in matching order

## Key terms

**Congruent triangles** ($\triangle ABC \cong \triangle PQR$): the same shape and size; corresponding sides and angles equal.

**Similar figures**: the same shape; smaller, larger or of equal size.

**Similar triangles** ($\triangle ABC \sim \triangle PQR$): corresponding angles equal, corresponding sides proportional.

**Corresponding sides** (similar triangles): the sides opposite equal angles.

**Scale factor**: the number that turns one triangle's sides into the other's.

**Axiom**: a fact we accept and build on, without proving it from anything simpler.

## The five tests of congruence

| Test | What must match |
|---|---|
| SSS | three sides |
| SAS | two sides and the angle **between** them |
| ASA | two angles and the side **between** them |
| AAS | two angles and a side not between them, opposite the same angle in both |
| RHS | a right angle, the hypotenuse and a side |

- any one of these makes the other parts equal too
- three well-chosen parts are enough; not all six
- AAS works: two angles match → the third matches → ASA

**Not tests:**

- **AAA:** the same shape, not the same size
- **SSA:** two sides and an angle not between them: the side opposite can swing to two places, two different triangles

## Order of the letters

- $\triangle ABC \cong \triangle PQR$: $A$ with $P$, $B$ with $Q$, $C$ with $R$
- the order is a promise: first letters partners, second, third
- $A$ matches $R$: write $\triangle ABC \cong \triangle RQP$ or the like
- corresponding sides: $AB$, $PQ$; $BC$, $QR$; $AC$, $PR$

## Writing a proof

1. name the two triangles
2. list three pairs of equal parts, each with its reason; mark (S), (A)
3. name the test; write the letters in matching order

**Parallelogram** $PQRS$, **diagonal** $PR$:

| Statements | Reasons |
|---|---|
| $PQ = RS$ (S) | opposite sides of a parallelogram |
| $QR = SP$ (S) | opposite sides of a parallelogram |
| $PR = RP$ (S) | common side |
| $\triangle PQR \cong \triangle RSP$ | SSS |

- a diagonal cuts a parallelogram into two equal halves, each half its area
- read the marks: ticks (sides), arcs (angles), right-angle squares
- the **common side**: a side both triangles share; easy to forget
- **a part to add:** two angles equal → the side between them (ASA); a right angle and a side → the hypotenuses (RHS)

- **with $x$:** congruent, so corresponding sides are equal: $1.7x - 1.3 = 3x - 3.9$, $x = 2$

## Similar figures

- two circles: always similar; two squares too
- two rectangles need not be: a thin strip, a sheet of paper
- A4, A5: the same shape; long side $= \sqrt{2} \times$ short side
- congruent figures are similar too, factor 1 (ratio 1 : 1)

## Similar triangles

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

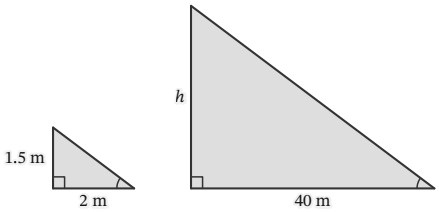
- find partners from the **equal angles**, never from how the triangles are turned
- corresponding **angles** equal; corresponding **sides** in proportion, not equal
- a missing side: write the ratios, put in the numbers, solve
- scale factor $6/4 = 1.5$: every side $\times 1.5$
- two equal angles $\rightarrow$ the third matches too
- an angle with $x$: it equals its partner: $5y - 3 = 62$, $y = 13$
- sides in the ratio 1 : 1: congruent

## The Nyatapola by its shadow

- sun's rays parallel $\rightarrow$ stick and temple make right-angled triangles with two equal angles $\rightarrow$ similar
- stick 1.5 m, shadow 2 m; temple's shadow 40 m:

$$\frac{h}{1.5} = \frac{40}{2} \qquad h = 1.5 \times 20 = 30 \text{ m}$$

- shadow measured from the point right under the top: from the plinth's edge, add half the base's width
- sources give its height as 30 m or as 33 m




The stick's triangle and the temple's, not drawn to one scale.

## Where it is used

- **SSS:** three stiff bars can't change shape; a square frame can be pushed over; so trusses are built of triangles
- **similarity:** a pantograph copies a drawing at another size
- maps, plans, models: every length by one factor, every angle the same

### COMMON MISTAKES

✗ letters in the wrong order
✓ $P$ goes with $R$ when $PQ = RS$.

✗ the reason left out, or the common side forgotten
✓ a reason beside each statement.

✗ SAS when the angle isn't between the sides
✓ it must be between.

✗ RHS with no right angle or hypotenuse known
✓ check both first.

✗ corresponding sides of similar triangles set equal
✓ in proportion.

✗ AAA or SSA given as a reason
✓ neither proves congruence.