Unit 14 · Plane Figures

STUDY NOTES

WHAT THE BLE ASKS

- **CDC's grid:** Geometry (Units 13, 14, 15, 16, 17, 18, 19, 20 together): 15 of 50 marks
- **In our 10 model papers:** 1-mark ×1 · 2-mark ×2 · 3-mark ×7
- **Model paper, Geometry part a (3 marks, application):** construct parallelogram $LMNO$: $LM = 8$ cm, $MN = 6$ cm, $\angle LMN = 75°$
- sketch first; leave every arc showing
- a paper may ask a triangle's angles, a kite's diagonals or a polygon's angles instead

Key terms

Vertex: a corner (plural vertices).

Base angles (isosceles): the two angles **opposite the equal sides**.

Quadrilateral: a figure with four straight sides.

Rhombus: all four sides equal.

Kite: two pairs of equal adjacent sides.

Trapezium: only one pair of parallel sides.

Construct: to draw exactly, from given measures, with a ruler and a compass.

Polygon: a closed figure with three or more straight sides.

Regular polygon: all sides equal **and** all interior angles equal.

Exterior angle: between a side carried on past a corner and the next side.

Formulas

$(n-2)\times 180°$	n sides; any polygon **Use:** sum of interior angles
$\dfrac{(n-2)\times 180°}{n}$	regular polygons only **Use:** each interior angle
$360°$	always **Use:** sum of exterior angles
$\dfrac{360°}{n}$	regular polygons only **Use:** each exterior angle
$180°$	interior + exterior **Use:** at each corner

- why $n-2$: from one corner, $n-2$ triangles of 180°
- test it: $n = 3$ gives 180°
- walk round a polygon: one full turn however many corners

Polygon	n	Sum	Each (regular)
Triangle	3	180°	60°
Square	4	360°	90°
Pentagon	5	540°	108°
Hexagon	6	720°	120°
Octagon	8	1080°	135°

Triangles

- interior angles add up to **180**°; use this first in almost every question
- equilateral: 60°, 60°, 60°
- isosceles: base angles equal, wherever they sit on the page
- right-angled isosceles: 90°, 45°, 45°; the longest side is the base
- set squares: 90°, 45°, 45° and 90°, 60°, 30°
- parallel lines in a figure: look for alternate angles too
- **exterior angle** (a side produced): = the two far interior angles added: $\angle QPR + \angle PQR = \angle PRS$
- two equal angles → the sides opposite them are equal (an isosceles triangle)
- any two sides together are longer than the third
- proof: $DAE \parallel BC$ through A: $\angle DAB = \angle B$, $\angle EAC = \angle C$ (alternate); the three at A make a straight line
- **verify** = test on real cases, by cutting, folding, measuring; a few cases can't show all

By sides		By angles	
equilateral	all equal	acute-angled	all under 90°
isosceles	two equal	right-angled	one 90°
scalene	none equal	obtuse-angled	one over 90°

Rhombus, kite, trapezium

Rhombus:

- four equal sides
- opposite sides parallel

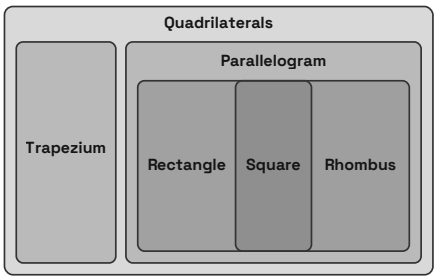
- opposite angles equal
- diagonals **bisect each other at right angles**
- diagonals bisect the corner angles
- every square is a rhombus; a rhombus is a square only if its angles are 90°

Kite:

- the longer diagonal bisects the shorter at right angles (it is the line of symmetry)
- the angles between the unequal sides are equal
- the shorter diagonal makes two isosceles triangles

Trapezium:

- co-interior angles along each non-parallel side add up to 180°



A box inside another: “is a kind of”. A trapezium stands apart: one pair of parallel sides.

A right angle where diagonals cross: a right-angled triangle, so Pythagoras' theorem finds a missing length

Constructions

Every time:

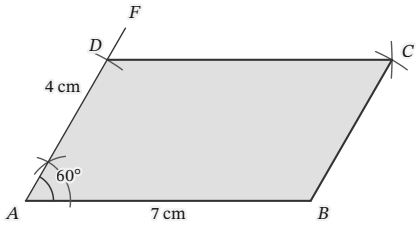
- rough sketch, corners in order, measures on it
- a compass angle of 60°, 90°, 120°, 45°, 30° or 75°
- don't rub out the arcs

Rectangle (two adjacent sides) / square (a side):

1. base line, mark AB
2. at A, 90°: $\angle FAB$; center A, the other side: cut AF at D
3. at B, 90°: $\angle HBA$; center B, the same radius: cut BH at C
4. join C, D

Parallelogram: opposite sides are equal. Three corners first, then two arcs find the fourth.

Given	Find C, D
a. two sides, angle between	angle at A; cut off AD; arcs from B (radius AD) and D (radius AB) meet at C
b. two sides, a diagonal	triangle ABC from arcs (diagonal from A, side from B); arcs from A, C meet at D
c. two sides, angle of side and diagonal	angle at A; arc from B cuts AF at C; arcs from A, C meet at D



Two adjacent sides and the angle between them: 7 cm, 4 cm, 60°.

- case *c*: the arc cuts AF twice; both make a parallelogram; use the farther point unless told otherwise
- a rectangle's diagonals are equal: 5 cm by 3 cm: both about 5.8 cm

Which tiles cover a floor?

- tiles close up round a corner only if a tile's angle **goes into 360° exactly**
- pentagon: 108°; three leave 36°, a fourth would overlap
- only 60°, 90°, 120° work: triangles, squares, hexagons
- past the hexagon: at most two fit round a corner; two can't close up
- mixing: octagons and squares: 135° + 135° + 90° = 360°

COMMON MISTAKES

✗ base angles mixed up when the triangle is turned
✓ opposite the equal sides.

✗ rectangles and squares not counted as parallelograms
✓ they are parallelograms too.

✗ construction without the shape's properties
✓ opposite sides must come out equal.

✗ arcs rubbed out, or no sketch
✓ keep both.

✗ $n \times 180°$ for the sum
✓ $(n - 2) \times 180°$.

✗ interior and exterior angles mixed up
✓ at each corner they add up to 180°.

✗ “each angle” formula on a shape that isn't regular
✓ regular polygons only.

✗ a half-diagonal used as a whole diagonal, or the reverse
✓ check which you have.

✗ a rhombus or rectangle called regular
✓ both conditions, or a square.