Unit 12 · Equation and Graph

STUDY NOTES

WHAT THE BLE ASKS

- **CDC's grid:** Algebra (Units 9, 10, 11, 12 together): 10 of 50 marks
- **In our 10 model papers:** 1-mark ×10 · 2-mark ×16
- **Model paper: 5 of Algebra's 10 marks**
  - **knowledge (1):** what two equations like $2x + y = 8$, $x + y = 5$ are called
  - **application (2):** solve them by a graph
  - **higher ability (2):** for what values of x is $x^2 - 5x + 6$ zero? (also Unit 10)

Key terms

Equation: says that two things are equal.

Variables: x, y: numbers that can change.

Linear equation: its graph is a straight line; two variables, many answers.

Simultaneous equations: two equations in the same two variables, true together.

Solution: the numbers that make both true.

Point of intersection: where the lines cross; the solution.

Degree: the highest power of the variable.

Quadratic equation: one variable, degree 2: $ax^2 + bx + c = 0$, $a \neq 0$.

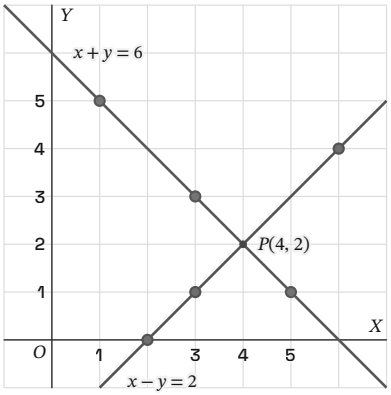
Roots: the values of the variable that make the equation true.

Formulas

$(x - a)(x - b) = 0$	product of factors is zero **Use:** then $x = a$ or $x = b$
$(x - p)(x - q) = 0$	roots p, q **Use:** building the equation from its roots

Solving by a graph

1. get one letter alone
2. three values of one letter; a table for each equation (two points fix a line; the third catches a slip)
3. choose a scale; plot both tables on one graph; draw two straight lines
4. read the crossing: first number x, second y
5. check the numbers in **both** equations



$x + y = 6$ and $x - y = 2$ cross at (4, 2): Asmi 4 mangoes, Anisha 2.

- a table: $x + y = 6 \rightarrow y = 6 - x$; $x = 1, 2, 3 \rightarrow y = 5, 4, 3$
- axes first, the same step all the way; one square for 5 years: 40 years is 8 squares
- a graph shows what the equation allows; the story decides what makes sense
- a crossing between grid lines can only be estimated: $x + y = 3$, $x - y = 2$: (2.5, 0.5)
- algebra pins it down exactly, in Grade 9

Lines	Example	Solutions
cross once	$x + y = 6$, $x - y = 2$	one
parallel	$x + y = 3$, $x + y = 5$	none
the same line	$x + y = 3$, $2x + 2y = 6$	endless

Word problems

1. a letter for each unknown; say what each stands for
2. one equation for each fact
3. tables of values; plot; draw the lines

4. the crossing is the answer

- "3 times the smaller": $x = 3y$; "3 more than the smaller": $x = y + 3$
- "3 times more" is unclear; in an exam nearly always $x = 3y$
- ages: "two years ago" takes 2 from **each** age; "after five years" adds 5 to each
- bills: $2x + y = 110$, $x + 2y = 85 \rightarrow (45, 20)$: notebook Rs 45, pen Rs 20
- heads and legs: x cats, y chickens: $x + y = 10$, $4x + 2y = 26 \rightarrow 3$ cats, 7 chickens
- each line is every pair one bill allows; only the crossing fits both

Quadratic equations

1. get **0** on one side
2. factorize: two numbers with product ac, sum b
3. set each factor equal to 0
4. solve each; write both roots
5. does each root make sense in the story?

- **if a product is 0, at least one factor is 0**
- $x^2 - 2x - 3 = 0$: $-3, +1$: $(x - 3)(x + 1) = 0$: $x = 3$ or -1
- $x^2 - 25 = 0$: $(x + 5)(x - 5) = 0$: $x = -5$ or 5
- $x^2 - 3x = 0$: $x(x - 3) = 0$: $x = 0$ or 3
- a squared bracket: $(x + 1)^2 - 2^2 = 0 \rightarrow (x + 3)(x - 1) = 0$: $x = -3$ or 1
- a linear equation, $x - 4 = 0$, has one root
- zero is special: $(x - 2)(x - 3) = 6$ does **not** give $x - 2 = 6$
- roots 2, 3: $(x - 2)(x - 3) = 0 \rightarrow x^2 - 5x + 6 = 0$
- graph of $y = x^2 - 2x - 3$: a curve; roots = where it meets the x-axis
- a curve crosses twice, touches once (one repeated root) or misses

In a story:

- bed 40 m^2, 3 m longer than wide: $w(w + 3) = 40$
- $w^2 + 3w - 40 = 0$: 8, -5: $(w + 8)(w - 5) = 0$
- $w = -8$ or 5: only 5 m wide, 8 m long makes sense
- ball: $5t(2 - t) = 0$: both roots mean something (leaves, comes back)

Where it is used

- crossing lines answer "when are these two equal?"
- **break-even:** total cost = total sales; before it a loss, after it a profit
- stall (made-up numbers): costs $y = 3000 + 60x$, sales $y = 100x$: $x = 75$ plates, both Rs 7,500
- market prices; comparing two plans: the crossing says when to change

COMMON MISTAKES

✗ a line through only two points
✓ a third catches a slip.

✗ axis numbers not evenly spaced
✓ the same step all the way.

✗ the crossing read as (y, x)
✓ x first.

✗ facts swapped in a word problem
✓ say what x and y stand for first.

✗ one root given, then stopped
✓ give both.

✗ dividing by x
✓ it loses the root 0; take x out as a factor.

✗ $x^2 = 49$ with one root
✓ two roots, ± 7.