# Unit 11 · Algebraic Fraction

STUDY NOTES

**WHAT THE BLE ASKS**

- **CDC's grid:** Algebra (Units 9, 10, 11, 12 together): 10 of 50 marks
- **In our 10 model papers:** 1-mark ×3 · 2-mark ×8
- **Model paper, Algebra part b (2 marks, understanding):** simplify $x/(x-y) - x/(x+y)$
- 1 mark: LCM $(x-y)(x+y)$, one fraction
- 1 mark: brackets opened: $2xy/(x^2-y^2)$
- a paper may ask it differently

## Key terms

**Algebraic fraction**: numerator and denominator are algebraic expressions.

**Lowest terms**: no common factor left to cancel.

**Like fractions**: the same denominator. **Unlike fractions**: different denominators.

**Reciprocal**: the fraction turned upside down: of $a/b$, $b/a$; a fraction × its reciprocal = 1.

## Rules

| | |
|---|---|
| $\frac{a}{x} + \frac{b}{x} = \frac{a+b}{x}$ | keep the denominator<br>**Use:** like fractions |
| $\frac{a}{x} + \frac{b}{y} = \frac{ay+bx}{xy}$ | pieces of the same size<br>**Use:** unlike fractions |
| $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ | tops together, bottoms together<br>**Use:** multiplying |
| $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ | × the reciprocal of the second<br>**Use:** dividing |

- an algebraic fraction follows every rule an ordinary fraction does
- no value where the denominator is 0: $x/(x+1)$ at $x = -1$
- we take it that the letters never make a denominator 0

## Factorizing patterns (Unit 10)

| Pattern | Example |
|---|---|
| Common factor | $x^2 + 3x = x(x+3)$ |
| Difference of squares | $x^2 - 9 = (x+3)(x-3)$ |
| Perfect square | $x^2 + 6x + 9 = (x+3)^2$ |
| Splitting the middle term | $x^2 - 5x + 6 = (x-3)(x-2)$ |
| Grouping | $xy + 2x + 3y + 6 = (x+3)(y+2)$ |

## Lowest terms

1. factorize the numerator and the denominator
2. cancel the common **factors**

- cancel only what **multiplies** the whole numerator and the whole denominator
- $(x+6)/(x+3)$ is not 2: at $x = 1$ it is 7/4
- terms joined by + or − can't be canceled
- check with one number: a few seconds, catches most mistakes

## Same denominator

1. keep the denominator; add or subtract the numerators
2. after a minus sign: a bracket round the numerator
3. factorize the new numerator; cancel if anything cancels

- $3x + 1 - (x+1) = 2x$; not $3x + 1 - x + 1$
- **turned round:** $4 - p = -(p-4)$, so $(p-4)/(4-p) = -1$, not 1

## Different denominators

1. factorize the denominators; find their LCM
2. multiply each fraction's top and bottom by what its denominator lacks
3. join the numerators, a bracket after a minus sign
4. simplify the numerator, factorize it, cancel

- the LCM is the smallest denominator that will do: $1/4 + 1/6$: 12, not 24
- the whole of the other denominator works too, but the working is much longer

## Multiplying and dividing

1. factorize everything first
2. to divide: turn over only the fraction **after** the ÷ sign
3. cancel across the whole product
4. only then multiply what is left

- $3/4 \times 2/3$ as an area: 12 parts, 3 columns by 2 rows = 6 parts
- $3 \div 1/2 = 3 \times 2 = 6$: ÷ a number less than 1 gives more
- never multiply out first: you would have to factorize it again

## Pipes add their rates

- a pipe filling in $a$ hours fills $1/a$ of the tank each hour
- two pipes in one hour:

$$\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$$

- time = work ÷ work done each hour:

$$1 \div \frac{a+b}{ab} = \frac{ab}{a+b}$$

- 3 and 6 hours: 18/9 = 2 hours, not the average 4½
- always less than the faster pipe alone
- **a leak subtracts:** $1/3 - 1/6 = 1/6$ of the tank each hour → 6 hours; time $ab/(b-a)$
- at $b = a$ the denominator is 0: the tank never fills
- Bhāskara's four fountains: 1, 2, 3, 6 cisterns a day = 12 a day → 1/12 of a day = 2 hours

## Where it is used

- two resistors in parallel: $R_1 R_2/(R_1 + R_2)$: 3 Ω, 6 Ω make 2 Ω
- thin lens: two fractions add to 1 over the focal length

### COMMON MISTAKES

✗ canceling terms: $(x+6)/(x+3) = 2$
✓ cancel factors only.

✗ the bracket forgotten after a minus sign
✓ the minus takes the whole numerator.

✗ $1/a + 1/b = 2/(a+b)$
✓ $(a+b)/(ab)$.

✗ the wrong fraction turned over
✓ only the one after ÷.

✗ $(a+b)^2 = a^2 + b^2$; $-(a-b) = a+b$
✓ open brackets with care.

✗ stopped too early
✓ look for a factor to cancel at the end.