Unit 10 · Algebraic Expression

STUDY NOTES

WHAT THE BLE ASKS

- **CDC's grid:** Algebra (Units 9, 10, 11, 12 together): 10 of 50 marks
- **In our 10 model papers:** 1-mark ×1 · 2-mark ×12
- **Model paper, Algebra (4 marks), this unit's question**
  - **a. application (2):** HCF of $x^2 - 5x + 6$ and $x^2 - 9$: factorize, then the shared factor: $x - 3$
  - **b. higher ability (2):** the values of x that make $x^2 - 5x + 6$ zero: $x = 2, x = 3$
- a paper may ask an LCM or a plain factorization instead

Key terms

Factors: expressions that divide a given expression exactly.

Factorization: writing an expression as a product of its factors.

Common factor: what every term shares; what is left is the **remaining factor**.

Trinomial: an expression of three terms.

Perfect square: a trinomial with a square at each end and a middle term twice the product of what was squared.

Coefficient: the number in front of x.

HCF: the product of the factors the expressions all share; it divides each exactly.

LCM: the smallest expression each divides exactly: shared factors × all the remaining factors.

Formulas

$a^2 - b^2 = (a + b)(a - b)$	one square minus another **Use:** difference of two squares
$a^2 + 2ab + b^2 = (a + b)^2$	middle term $2ab$ **Use:** perfect square
$a^2 - 2ab + b^2 = (a - b)^2$	middle term $-2ab$ **Use:** perfect square
$ax^2 + bx + c$	two numbers: product ac; sum or difference b **Use:** splitting the middle term
$\text{HCF} \times \text{LCM} = A \times B$	for two expressions A, B **Use:** a check

Which method?

1. always a **common factor** first
2. two terms: a **difference of two squares**?
3. three terms: a **perfect square**? else split the middle term
4. four terms: **group**, or make a perfect square, then a difference of squares
5. check by multiplying back, or one number in both forms

Common factor and grouping

- look for what every term shares; write it once outside a bracket
- $6 \times 47 + 6 \times 53 = 6(47 + 53) = 600$
- groups must leave the **same bracket**, which comes out in its turn:

$$2xy + 3 + 6x + y = 2x(y + 3) + 1(y + 3)$$
$$= (y + 3)(2x + 1)$$

- turn a bracket round: $-y(q - p) = +y(p - q)$
- so $x(p - q) - y(q - p) = (p - q)(x + y)$

Difference of two squares

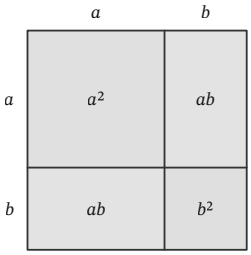
- $25x^2 = (5x)^2$, not $(25x)^2$: root the number too
- a **sum** of two squares has no factors of this kind
- arithmetic: $13 \times 7 = (10 + 3)(10 - 3) = 91$
- $97 \times 103 = 100^2 - 3^2 = 9991$
- shaded areas: square garden x m with a 6 m pond: $x^2 - 36 = (x + 6)(x - 6)$; a ring $\pi(R + r)(R - r)$

Perfect squares

- $x^2 + 6x + 9$: yes, $2 \times x \times 3 = 6x$; $x^2 + 5x + 9$: no

- **fill the blank:** twice the product of the two roots: $x^2 + \ldots + 16 \to 2 \times x \times 4 = 8x$; $9a^2 - \ldots + 49b^2 \to 42ab$
- $(3 + 4)^2 = 49$; $3^2 + 4^2 = 25$: the missing 24 is $2ab$, two rectangles
- four terms: $16x^2 + 40x + 25 - y^2 = (4x + 5)^2 - y^2$
- then $(4x + 5 + y)(4x + 5 - y)$
- the square after a minus:

$$4a^2 - b^2 + 20b - 100 = (2a)^2 - (b^2 - 20b + 100)$$
$$= (2a)^2 - (b - 10)^2$$



A square of side $a + b$: a^2, b^2 and **two** rectangles ab.

Splitting the middle term

1. multiply a by c
2. two numbers with that product: their **sum** is b if ac is positive; their **difference** if negative
3. split the middle term with them
4. a common factor from each pair; the brackets match; take that out

Expression	The two numbers	$b = 5, c = 6$
$x^2 + bx + c$	both positive	$+3, +2$
$x^2 - bx + c$	both negative	$-3, -2$
$x^2 + bx - c$	bigger positive	$+6, -1$
$x^2 - bx - c$	bigger negative	$-6, +1$

- $x^2 + 4x + 3 = x^2 + x + 3x + 3 = (x + 1)(x + 3)$
- $a \neq 1$: $3x^2 + 11x + 6$: $3 \times 6 = 18$; 9 and 2:
- $3x(x + 3) + 2(x + 3) = (x + 3)(3x + 2)$
- most expressions don't factorize so: $x^2 + x + 1$: no whole numbers
- a bracket as one letter: $(a + b)^2 + 11(a + b) + 18$: 9 and 2 $\to (a + b + 9)(a + b + 2)$
- x^2 last and negative: $35 - 2x - x^2 = 7(5 - x) + x(5 - x) = (5 - x)(7 + x)$

HCF and LCM

HCF: 1. factorize each; 2. multiply the factors they share

LCM: 1. factorize each; 2. shared factors × all the remaining; a shared factor **once**

- $A = x^2 + x - 20 = (x + 5)(x - 4)$, $B = x^2 - 25 = (x + 5)(x - 5)$
- HCF $= x + 5$; LCM $= (x + 5)(x - 4)(x - 5)$
- "highest" = the highest power of the letters; not a number from values: $x^2 - 4$, $x^2 + 4x + 4$: HCF $x + 2$

A product is zero

- **only when one of its factors is zero**
- $(x - 2)(x - 3) = 0 \to x = 2$ or $x = 3$
- ball: $h = 20t - 5t^2 = 5t(4 - t)$: $h = 0$ at $t = 0$ (leaves the hand), $t = 4$ (back)
- 15 m up: $5(t - 1)(t - 3) = 0$: $t = 1$ going up, $t = 3$ coming down
- RSA: multiplying two huge primes is quick; getting them back from the product is not

COMMON MISTAKES

✗ $(a + b)^2 = a^2 + b^2$
✓ $a^2 + 2ab + b^2$; $(a - b)^2$ likewise.

✗ $4x^2$ as $(4x)^2$
✓ $(2x)^2$.

✗ $-(a - b) = a + b$
✓ $-a + b$.

✗ the common factor forgotten
✓ take it out first, or the answer can still be factorized.

✗ sum where the sign calls for difference
✓ look at the sign of c.

✗ a shared factor counted twice in an LCM
✓ once.