Unit 9 · Indices

STUDY NOTES

WHAT THE BLE ASKS

- **CDC's grid:** Algebra (Units 9, 10, 11, 12 together): 10 of 50 marks
- **In our 10 model papers:** 1-mark ×6 · 2-mark ×4
- **Model paper, Algebra part a (1 mark, knowledge):** express x^m / x^n as a power of x: x^{m-n}
- the grid wants every topic of an area in proportion: a paper may word it differently, or ask a little more

Key terms

Base a in a^n: what is multiplied.

Index (exponent) n: how many of it are multiplied; a count.

Power: the whole thing, a^n; 2^6 is "2 to the power 6".

Laws of indices

$x^m \times x^n = x^{m+n}$	same base: add the indices **Use:** multiplying
$x^m \div x^n = x^{m-n}$	same base, $x \neq 0$: subtract **Use:** dividing
$x^0 = 1$	$x \neq 0$ **Use:** zero index
$x^{-m} = \frac{1}{x^m}$	$x \neq 0$; also $\frac{1}{x^{-m}} = x^m$ **Use:** negative index
$(xy)^m = x^m y^m$	the index goes to each **Use:** power of a product
$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$	the index goes to each **Use:** power of a quotient
$(x^m)^n = x^{mn}$	multiply the indices **Use:** power of a power

Where each law comes from

- write the powers out and count: $x^2 \times x^3 = (x \times x) \times (x \times x \times x) = x^5$
- $x^5 \div x^2$: each x below cancels one above: x^3
- $x^3 \div x^3 = 1$ and $= x^{3-3} = x^0$, so $x^0 = 1$
- $x^m \times x^{-m} = x^0 = 1$, so $x^{-m} = 1/x^m$
- going down the powers of 2: each step halves; no reason to stop at 1
- $(x^2)^3 = x^2 \times x^2 \times x^2 = x^6$
- $x \neq 0$: the steps divide by x^m; $0 \div 0$ has no meaning

Reading an index

- an index belongs to what it touches; with a bracket, the whole bracket
- $(-3)^2 = 9$; $-3^2 = -9$
- even index: a negative base turns positive: $(-2x)^4 = 16x^4$
- odd index: stays negative: $(-2x)^3 = -8x^3$
- $(3a)^0 = 1$, for $a \neq 0$; $3a^0 = 3 \times 1 = 3$
- a number in the bracket takes the index: $(3x^3)^2 = 9x^6$
- $(2x)^2 = 4x^2$: twice the side, 4 times the area
- $x^2 = x \times x$; $2x = x + x$: $x = 5$ gives 25, 10
- negative index = "turn it over": $2^{-3} = 1/8$, not -8

Add or multiply?

Form	Do	Example
side by side	**add** the indices	$x^2 \times x^3 = x^5$
bracket, index outside	**multiply** them	$(x^2)^3 = x^6$

- unsure? write them out and count
- the laws are for × and ÷ only: $(x+y)^2$ is **not** $x^2 + y^2$ ($x = 2$, $y = 3$: 25, not 13)

Simplifying: the patterns

1. numbers in front: multiply or divide as usual; only the indices add or subtract
2. letters in the index: same laws
3. a minus before a bracket: keep it, then change **every** sign:
$$x^a \div x^{b-c} = x^{a-(b-c)} = x^{a-b+c}$$
4. indices that cancel:
$$x^{a-b} \times x^{b-c} \times x^{c-a} = x^0 = 1$$
5. squares in the index: $(x^{a-b})^{a+b} = x^{a^2-b^2}$; the three brackets add to $x^0 = 1$

6. numbers as powers of one base: $4 = 2^2$, $8 = 2^3$, $25 = 5^2$, $125 = 5^3$:
$$\frac{5^3 \times 125^3}{25^3} = 5^{3+9-6} = 5^6$$
7. a missing index (fill the box): same base on both sides, then the indices are equal: $4^n = 8^2 = 64 = 4^3$ $\rightarrow$ the box holds $n = 3$
8. values: bases different, so the laws can't join them: work each out, multiply:
$a = 1, b = 2, c = 3$: $\quad a^b \times b^c \times c^a = 1 \times 8 \times 3 = 24$

The rice pot: indices count doublings

Minutes	0	20	40	60	120
Doublings	0	1	2	3	6
Bacteria	1	2	4	8	64

- bacteria can double in as little as 20 minutes in warm food
- 2 hours: $2^6 = 64$
- 10 hours: 30 doublings: $2^{30} = (2^{10})^3 \approx 10^9$; exactly 1,07,37,41,824
- times add, so indices add: $2^6 \times 2^3 = 2^9$
- how many times as many: $2^9 \div 2^3 = 2^6$
- 2^0: no doubling yet, still 1
- negative: 20 minutes ago: $64 \times 2^{-1} = 32$
- doubling every 40 minutes: 2 hours $= 2^3 = 8$, an eighth, not half: a change in the **index** counts far more

Where it is used

- food safety: never over 2 hours out of the refrigerator (1 hour above 32 °C)
- one byte: $2^8 = 256$ values; three bytes: $(2^8)^3 = 2^{24}$ colors
- earthquakes: each whole step ≈ 32 times the energy; two steps $32^2 = 1024$
- $10^{-3} = 1/1000$: scientific notation, Unit 3

COMMON MISTAKES

✗ x^2 written as $2x$
✓ $x \times x$.

✗ $x^3 \times x^2 = x^6$
✓ add: x^5.

✗ $(3x^3)^2 = 3x^6$
✓ $9x^6$: the number takes the index.

✗ $-y^2$ treated as $(-y) \times (-y)$
✓ only $(-y)^2$ means that.

✗ x^{-m} written as $-x^m$
✓ $1/x^m$.

✗ x^0 written as 0
✓ 1.

✗ a sign lost subtracting an index of two terms
✓ use a bracket.