Unit 8 · Area and Volume

STUDY NOTES

WHAT THE BLE ASKS

- **CDC's grid:** Mensuration: 5 of 50 marks
- **In our 10 model papers:** 1-mark ×30 · 2-mark ×10
- **Model paper: one 5-mark question** (a trapezium of land with a round well)
  - **knowledge (1):** a formula
  - **understanding (1):** the well's area
  - **application (2):** the land left over
  - **higher ability (1):** is a length of wire enough to go round?
- use the π the question gives; a unit, m^2 or cm^2, with every area

Key terms

Area: the amount of flat surface a shape covers, in squares of a chosen size.

Height of a triangle: from a vertex straight down, at a right angle, to the opposite side; any side can be the base.

Trapezium: a quadrilateral with one pair of parallel sides.

Circumference: the whole length of a circle's boundary.

Pi (π): circumference ÷ diameter, the same for every circle; 22/7 or 3.14, neither exact.

Formulas

$A = l \times b,\ P = 2(l + b)$	length, breadth **Use:** rectangle
$A = a^2 = \frac{d^2}{2},\ P = 4a$	side a; diagonal d (a square is a rhombus) **Use:** square
$A = \frac{1}{2} \times b \times h$	any triangle; h at a right angle to b **Use:** triangle
$A = \frac{1}{2} \times b \times p$	b, p at the right angle; equal: $p^2/2$ **Use:** right-angled triangle
$A = \frac{\sqrt{3}}{4} a^2$	side a; height $\sqrt{3}a/2$ **Use:** equilateral triangle
$A = \frac{b}{4}\sqrt{4a^2 - b^2}$	a equal side, b base **Use:** isosceles triangle
$A = b \times h$	h between the parallel sides **Use:** parallelogram
$A = \frac{1}{2}\, d_1\, d_2$	diagonals, at right angles **Use:** rhombus, kite
$A = \frac{1}{2}\, h\, (a + b)$	a, b the parallel sides **Use:** trapezium
$A = \frac{1}{2}\, d\, (h_1 + h_2)$	d diagonal; h_1, h_2 perpendiculars onto it **Use:** any quadrilateral
$C = 2\pi r = \pi d$	a length, in m **Use:** circumference
$A = \pi r^2 = \frac{1}{4}\pi d^2$	an area, in m^2 **Use:** circle

Perimeter

- **perimeter** = all the sides added; a dotted height is **not** a side
- a side missing in a right-angled triangle: Pythagoras first; 3, 4 → 5
- back from a perimeter: square side = $P \div 4$; rectangle $2(15 + b) = 54 \rightarrow b = 12$
- length twice the breadth: x, $2x$; $2(2x + x) = 60$

Why each one works

- cut and move the pieces: nothing added, nothing lost, the area stays
- right-angled triangle: half a rectangle
- parallelogram: cut, move → a rectangle
- rhombus, kite: two triangles on one diagonal; the other is their heights added
- trapezium: a diagonal → two triangles of height h, bases a, b
- any quadrilateral: a diagonal → two triangles
- circle: thin slices head to tail → a rectangle, height r, base $\pi\, r$
- forget a formula: isosceles or equilateral → the height by Pythagoras, then ½ × base × height
- equilateral: $AD = \sqrt{a^2 - (a/2)^2} = \sqrt{3}a/2$

Heights

- right-angled: the height is the other side at the right angle; never the hypotenuse
- a leaning triangle: the height falls outside, on the base carried on; the formula still works
- parallelogram: the straight distance between the parallel sides, not the slanting side
- tilting one (same base, height): same area, longer sides
- trapezium: parallel sides may stand upright, its height then runs across; look for the arrows
- rhombus: $b \times h$ works too

Shapes not on the list

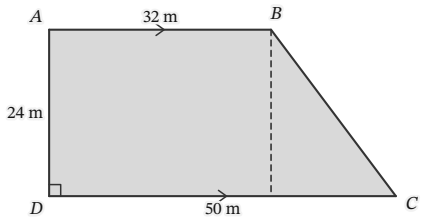
- **joined shape:** cut it into known shapes, add; cut another way as a check: the same area
- **shaded part:** whole shape − the part left out
- **working back:** put in what is known, solve for the length
- **cost** = area × rate
- **how many** boards, tiles = area ÷ one's area; or area × tiles per m^2

Circle

- given a diameter: halve it before you square it
- from a circumference: find r first
- from an area: find r^2, then its square root
- no π given: say which you took
- **double the radius:** area ×4 ($\pi\,(2r)^2 = 4\pi\, r^2$); circumference ×2

Suraksha's field

- trapezium, parallel sides 30 m, 60 m, 30 m apart: ½ × 30 × (30 + 60) = 1,350 m^2 ≈ 2.65 ropani
- well 1.4 m across: r = 0.7 m; lid 22/7 × 0.7 × 0.7 = 1.54 m^2; rim 4.4 m
- twice as wide: lid 6.16 m^2 (four times), rim 8.8 m (double)
- ropani ≈ 508.72 m^2; bigha ≈ 6,772.63 m^2



A trapezium with a right angle at D: the dotted line cuts 18 m off DC, so the slant side comes from Pythagoras' theorem.

COMMON MISTAKES

✗ radius where a diameter is given, or the reverse
✓ halve the diameter first.

✗ a slanting side as the height
✓ height at a right angle to the base.

✗ cm for an area, or no unit
✓ cm^2, m^2.

✗ the 1/2 forgotten
✓ triangle, rhombus, trapezium.

✗ trapezium's perimeter with a side missing
✓ Pythagoras' theorem first.

✗ $2\pi\, r$, $\pi\, r^2$ mixed up
✓ the 2 in r^2 goes with the square in m^2.