Unit 4 · Ratio and Proportion

STUDY NOTES

WHAT THE BLE ASKS

- **CDC's grid:** Arithmetic (Units 2, 3, 4, 5, 6, 7 together): 14 of 50 marks
- **In our 10 model papers:** 1-mark ×10 · 2-mark ×4
- **Model paper, Arithmetic part c (2 marks, higher ability):** share Rs 1,00,000 in the ratio of two ages; how much more does the elder get?
- 12 : 8 = 3 : 2 → 5 parts of Rs 20,000 → Rs 60,000, Rs 40,000 → **Rs 20,000 more**
- read the end of the question: "how much more" is one step past the shares

Key terms

Ratio $a : b$: a/b, for two quantities in the same unit; read "a is to b".

Lowest terms: no common factor but 1: $10 : 25 = 2 : 5$.

Proportion $a : b :: c : d$: two equal ratios; read "a is to b as c is to d"; $a : b = c : d$ too.

Extremes: the outer numbers, a and d.

Means: the inner numbers, b and c.

Continued proportion: the two means are the same number: 4 : 6 :: 6 : 9, so 4, 6, 9.

Scale of a map: a ratio, the same unit on both sides.

Formulas

$a \times d = b \times c$	extremes' product = means' product **Use:** test four numbers; find a missing one
ax, bx	amounts in the ratio $a : b$, a block x **Use:** unknown amounts, ages
$x =$ total $\div (a + b)$	one block **Use:** sharing a total

- why: $a/b = c/d$; $\times bd$ both sides → $a \times d = b \times c$
- 1 : 2 :: 2 : 4: $1 \times 4 = 2 \times 2$ ✓

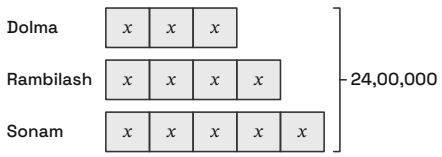
Ratio

- **subtracting** says how much more; **dividing** says how many times
- 48 kg, 32 kg: 16 kg heavier; 1.5 times as heavy ("1.5 times" means the same at any size)
- **order matters:** the first number belongs to the first thing named
- **no unit:** Rs 10 to Rs 25 is 2 : 5; the rupees cancel
- same unit first; 1 dozen to 6 is 12 to 6
- × or ÷ both numbers by the same number (not 0): the ratio stays
- **adding** the same number to both changes it (unless 1 : 1)
- 2 : 5: a pencil $2/5 = 0.4$ times a pen; a pen $5/2 = 2.5$ times a pencil
- a decimal ratio to whole numbers: $1 : 1.5 : 3 \times 2 = 2 : 3 : 6$
- sex ratio (2021 census): 95.59 males for every 100 females

A ratio is not always a fraction of the whole: 30 girls, 20 boys: girls : boys = 3 : 2; girls' part of the class = 30/50 = 3/5

Sharing a total

1. add the ratio's numbers
2. total ÷ that sum = one block
3. each share = its number × one block



Incomes in the ratio 3 : 4 : 5: twelve equal blocks make Rs 24,00,000, so one block is Rs 2,00,000.

- momo stall: Rs 20,000, 30,000, 50,000 → 2 : 3 : 5 → 10 blocks of Rs 6,000 → Rs 12,000, 18,000, 30,000
- each friend's profit = 60% of what they put in
- partners may agree on another ratio; maths carries out what was agreed
- dividends the same way: 150 shares, 50 shares: Rs 6,000, Rs 2,000 = 3 : 1

- "B twice A, C three times B" → A : B : C = 1 : 2 : 6, then share
- angles of a triangle in a ratio: share **180**°; 2 : 3 : 4 → $9x = 180°$, 40°, 60°, 80°

Ages that change

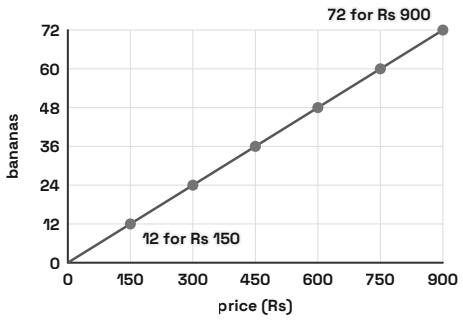
- ages 4 : 5 → $4x$ and $5x$
- add the years to the **ages**: $(4x + 4) : (5x + 4)$, never $(4 + 4) : (5 + 4)$
- set the new ratio equal, cross multiply, solve for x
- a number y added to both: $(4 + y) : (5 + y) = 5 : 6$ → $y = 1$; subtracted: $(24 - y) : (30 - y)$

Map scale

- 1 : 25,000: 1 cm on the map = 25,000 cm = 250 m on the ground
- any unit, the same on both sides
- 1 : 50,000: 4 cm is 2 km
- Nepal's Survey Department: Tarai, middle mountains 1 : 25,000; high hills, Himalaya 1 : 50,000

Proportion in use

- missing term: write the proportion, cross multiply, solve
- quantities in proportion → a **straight line through 0**: double one, the other doubles



12 bananas for Rs 150: every price and number of bananas on one straight line through 0.

- use it only when both really grow by the same factor: a child's height doesn't; wholesale prices bend the line
- more workers, fewer days: indirect variation, Unit 6
- **shadows:** sun's rays parallel → at one moment, height : shadow the same for every upright thing
- **concrete** M20: cement : sand : aggregate = 1 : 1.5 : 3; strength mainly depends on the mix ratio

COMMON MISTAKES

✗ different units left in
✓ kg with gm, hours with minutes, rupees with paisa: one unit first.

✗ order swapped
✓ the first number belongs to the first thing named.

✗ stopped at the shares
✓ "how much more" needs the difference.

✗ divided by one number of the ratio
✓ divide by their sum.

✗ years added to the ratio's numbers
✓ add them to the ages, $4x$ and $5x$.

✗ 3 : 2 read as 3/2 of the class
✓ a part with a part; compare with the whole.