Unit 3 · Rational and Irrational Numbers

STUDY NOTES

WHAT THE BLE ASKS

- **CDC's grid:** Arithmetic (Units 2, 3, 4, 5, 6, 7 together): 14 of 50 marks
- **In our 10 model papers:** 1-mark ×11 · 2-mark ×10
- **Arithmetic question, model paper**
  - **a. understanding (1):** a number in scientific notation: 37,200 liters = 3.72×10^4 liters
  - **c. understanding (2):** a recurring decimal to a fraction: $0.\overline{34} = 34/99$; the working must be shown

Key terms

Rational number: can be written as a/b, a and b integers, $b \neq 0$: 2, −7, 5/8, 2.13, $1.\overline{6}$.

Irrational number: can't be written that way: $\sqrt{2}$, $\sqrt{5}$, $\sqrt[3]{10}$, π.

Non-terminating, non-recurring decimal: never ends, no block repeats: irrational.

Real numbers R: the rational and the irrational numbers together.

Real line: the number line; every real number has a point, every point is a real number.

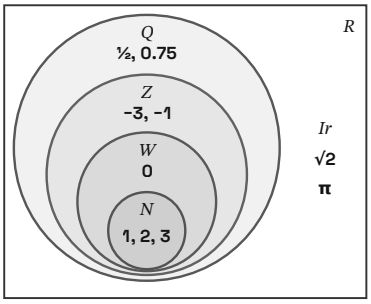
Scientific notation (standard form): one non-zero digit before the point × a power of 10.

Formulas

$a \times 10^n$	$1 \leq a < 10$, n an integer **Use:** scientific notation
$x^m \times x^n = x^{m+n}$	powers add **Use:** multiplying
$x^m / x^n = x^{m-n}$	powers subtract **Use:** dividing
$h = \sqrt{p^2 + b^2}$	Pythagoras theorem **Use:** roots on the number line

Kinds of number

- $N = \{1, 2, 3, \ldots\}$ natural
- $W = \{0, 1, 2, 3, \ldots\}$ whole
- $Z = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ integers
- Q rational; Ir irrational; R real
- $N \subseteq W \subseteq Z \subseteq Q \subseteq R$; $Ir \subseteq R$
- Q, Ir disjoint; their union is R



The real numbers as a Venn diagram: N inside W inside Z inside Q; Ir is the rest of R, apart from Q.

- **5**
  - natural
  - integer
  - real
  - whole
  - rational
- $\sqrt{5}$: irrational, real only

Decimal	Example	Number
terminating	2.13	rational
recurring	$0.\overline{142857}$	rational
non-terminating, non-recurring	$\sqrt{2}$, π	irrational

- "rational" from **ratio**, not "reasonable"
- not every root is irrational: $\sqrt{49} = 7$
- $\pi \approx 22/7$: 22/7 is rational, an **approximate value**: 3.142857... against 3.141592...

Recurring decimal to a fraction

1. let $x =$ the decimal (i)
2. multiply (i) by 10 if one digit repeats, 100 if two, 1000 if three (ii)
3. (ii) − (i): the tails cancel
4. solve for x

- $x = 0.333\ldots$; $10x = 3.333\ldots$; $9x = 3$; $x = 1/3$
- $x = 0.4141\ldots$; $100x = 41.4141\ldots$; $99x = 41$; $x = 41/99$
- a block of n digits goes over n nines

- keep a whole part: $1.\overline{57}$: $100x = 157.5757\ldots$, $99x = 156$, not 157

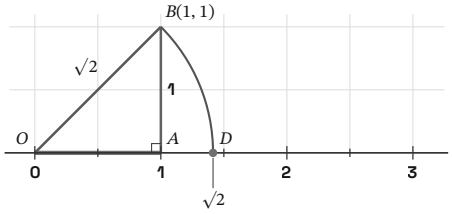
Roots on the number line

$\sqrt{2}$:

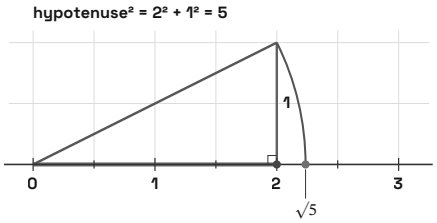
1. right-angled triangle OAB on the line: $OA = 1$, $AB = 1$, so $OB = \sqrt{2}$
2. compass on O, radius OB: arc to cut the line at D
3. D is the point $\sqrt{2}$, between 1.4 and 1.5

$\sqrt{3}$: at the point $\sqrt{2}$ an upright 1 unit tall; $h = \sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{3}$; arc from 0 down to the line

- base $\sqrt{k}$, upright 1 $\rightarrow$ hypotenuse $\sqrt{k+1}$
- quicker: $\sqrt{5}$: base 2, upright 1 ($2^2 + 1^2 = 5$); $\sqrt{8}$: base 2, upright 2



OB is the hypotenuse of a triangle with sides 1 and 1; the arc about O brings its length down to D.



$\sqrt{5}$ (Ex 3.1 Q4): base 2, upright 1, the hypotenuse swung down from 0.

Scientific notation

- $1673 = 1.673 \times 10^3$; $0.00157 = 1.57 \times 10^{-3}$
- point moved **left**: positive power (big numbers)
- point moved **right**: negative power (numbers less than 1)
- $10^{-3} = 1/10^3$; $10^0 = 1$, so $1.76 \times 10^0 = 1.76$
- $340 = 3.4 \times 10^2$ only; 0.34×10^3, 34×10^1 have the value but not the form
- calculators: $1.496\text{E}8 = 1.496 \times 10^8$

Working with it

Operation	What to do
add, subtract	same power first, then the numbers in front
multiply	multiply the numbers, add the powers
divide	divide the numbers, subtract the powers

- finish with one non-zero digit before the point
- $3.4 \times 10^2 + 4.57 \times 10^3$: $3.4 \times 10^2 = 0.34 \times 10^3$, so $(0.34 + 4.57) \times 10^3 = 4.91 \times 10^3$
- like money: 3 hundreds and 4 thousands are not 7 of anything

Where it is used

- A paper: long side = short side $\times \sqrt{2}$; fold in half, same shape
- $L/1 = 1/(L \div 2) \rightarrow L^2 = 2$; A4: $297 \div 210 = 1.41428\ldots$, against $1.41421\ldots$
- $\sqrt{2}$ isn't a fraction: whole millimeters only come near
- red blood cells: 4.2×10^6 to 5.7×10^6 per microliter (men)

COMMON MISTAKES

✗ every long decimal called irrational
✓ $0.\overline{142857}$ repeats: rational.

✗ every root called irrational
✓ $\sqrt{49} = 7$.

✗ $\times 10$ when two digits repeat
✓ count the digits under the bar: $\times 100$.

✗ "Let $x =$" and the subtraction left out
✓ a 2-mark question needs the working.

✗ a power with the wrong sign
✓ a number less than 1 has a negative power.

✗ numbers in front added before the powers match
✓ same power first.

✗ $2 \times 10^3 + 3 \times 10^3 = 5 \times 10^6$
✓ 5×10^3: powers add only when you multiply.

✗ stopped at 10.53×10^6 or 0.4×10^{-5}
✓ 1.053×10^7, 4×10^{-6}.