Unit 2 · Whole Numbers

STUDY NOTES

WHAT THE BLE ASKS

- **CDC's grid:** Arithmetic (Units 2, 3, 4, 5, 6, 7 together): 14 of 50 marks
- **In our 10 model papers:** 1-mark ×10 · 2-mark ×2
- **Model paper, Arithmetic part d (1 mark, higher ability):** shaded box = 1, empty box = 0; shade six boxes to show 43 in binary
- 43 = 101011_2: shaded, empty, shaded, empty, shaded, shaded
- ladder on rough paper; check it before you shade

Key terms

Whole numbers: 0, 1, 2, 3, ...: the counting numbers with 0 added.

Base: how many make a bundle; the number of digits a system uses.

Decimal system (Hindu-Arabic): base 10; digits 0–9; places 10^0, 10^1, 10^2, ...

Binary system: base 2; digits 0, 1; places 2^0, 2^1, 2^2, ...

Quinary system: base 5; digits 0–4; places 5^0, 5^1, 5^2, ...

Expanded form: each digit × its place value, added.

Suffix: the small number after a number, its base: 101_2, 1204_5. No suffix = decimal.

Place values

Place	2^5	2^4	2^3	2^2	2^1	2^0
Binary	32	16	8	4	2	1

Place	5^5	5^4	5^3	5^2	5^1	5^0
Quinary	3125	625	125	25	5	1

- each place = base × the place on its right
- $10 \div 10 = 1 = 10^0$; the ones place fits the pattern
- $1256 = 1 \times 10^3 + 2 \times 10^2 + 5 \times 10^1 + 6 \times 10^0$

Counting

Decimal	Binary	Quinary
1	1	1
2	10	2
3	11	3
4	100	4
5	101	10
6	110	11
7	111	12
8	1000	13
9	1001	14
10	1010	20

- binary: 1 is already full, so the carry comes at once: after 1 comes 10, after 11 comes 100
- read digit by digit: 10_2 "one zero" = **two**; 100_2 "one zero zero" = four
- quinary: 0, 1, 2, 3, 4, 10, 11, ... 44, 100, 101, ...
- **the base itself is never a digit:** no 2 in binary, no 5 in quinary; 1256_5 isn't a number

Decimal to another base: the ladder

1. divide by the base; write the remainder at the right
2. divide the quotient the same way, until the **quotient is 0**
3. write the remainders **from bottom to top**

- remainders: 0 or 1 in base 2; 0 to 4 in base 5
- why: 27 pens in pairs → 13 pairs, 1 left: the ones digit; 13 pairs in twos → 6, 1 left: the twos digit...
- the first remainder is the **rightmost** digit

- the same ladder in base 10 gives back 1256: it pulls off a number's digits from the right

		Rem.	What the row means
2	27		
2	13	1	13 groups of 2, 1 left over
2	6	1	6 groups of 4, 1 group of 2 left
2	3	0	3 groups of 8, 0 groups of 4 left
2	1	1	1 group of 16, 1 group of 8 left
	0	1	0 groups of 32, 1 group of 16 left

27 divided by 2 again and again: read from the bottom up, $27 = 11011_2$.

		Rem.
5	432	
5	86	2
5	17	1
5	3	2
	0	3

The same ladder in base 5: remainders 0 to 4, read from the bottom up, $432 = 3212_5$.

For a small number, by powers:
$87 = 64 + 16 + 4 + 2 + 1 = 1010111_2$;
$179 = 125 + 2 \times 25 + 4 = 1204_5$

Another base to decimal

1. write it in expanded form in powers of the base
2. simplify and add

- $100101_2 = 2^5 + 2^2 + 2^0 = 32 + 4 + 1 = 37$
- binary: the zeros add nothing; add the place values under the 1s
- quinary: each digit must be **multiplied**: a 4 in the 625 place = 2500
- $142013_5 = 3125 + 2500 + 250 + 0 + 5 + 3 = 5883$

Between binary and quinary

- always **through decimal**: binary → decimal → quinary, or quinary → decimal → binary
- to compare 1101_2 and 24_5: change both to decimal first; $13 < 14$, difference 1 (as a decimal number)
- binary needs **more** digits: places double, quinary's grow five times; 7521: 6 digits in quinary, 13 in binary

Where it is used

- switch OFF = 0, ON = 1: computers store every number, photo, song in binary
- 87 = seven switches: ON, OFF, ON, OFF, ON, ON, ON
- n binary digits → 2^n patterns, so memory comes in 64, 128, 256, 512
- QR code: dark or light squares = binary you can see
- quinary: rarely used; tally marks are bundled in fives
- learning a second base shows what place value is: 10 is a choice

COMMON MISTAKES

✗ remainders read from the top down
✓ from the bottom up; else the number comes out backwards.

✗ the last step left out
✓ go on to $1 \div 2 = 0$ remainder 1: it is the first digit.

✗ 5 or more as a digit in base 5
✓ digits stop at 4.

✗ the suffix forgotten on the answer
✓ 11010_2, 1204_5.

✗ powers started at 2^1
✓ start at 2^0, or every place is worth double.

✗ 10_2 read as "ten"
✓ "one zero", which means two.

✗ two bases compared by their looks
✓ change both to decimal first.