Unit 19 · Transformation

HARDER PRACTICE QUESTIONS

Full marks 35

Write your answers on separate paper. Show your working. Draw the axes on squared paper and plot every object and its image, one unit a square. Name each image with a dash: the image of A is A'.

1 Spot the errors [4]

Roshan wrote these four. Each one is wrong. Correct it, and say what went wrong.

i. The image of $(3, -5)$ in the X-axis is $(-3, -5)$.
ii. The image of $(0, 4)$ in the Y-axis is $(0, -4)$.
iii. In a reflection, every point of the figure moves the same distance.
iv. When a triangle is translated 3 cm to the right, each of its sides gets 3 cm longer.

2 One mirror, then the other [4]

$\triangle ABC$ has corners $A(2, 1)$, $B(5, 1)$ and $C(5, 4)$.

i. Draw it and its image $\triangle A'B'C'$ in the X-axis. Write the corners.
ii. Draw the image of $\triangle ABC$ in the Y-axis, and write its corners.
iii. Now reflect $\triangle A'B'C'$ in the Y-axis. Write the corners of this new image, $\triangle A''B''C''$.
iv. Is $\triangle A''B''C''$ the image of $\triangle ABC$ in an axis? Is it a translation of $\triangle ABC$? Give a reason for each.

3 Find the object, find the mirror [4]

i. $P'(-3, 7)$ is the image of P in the Y-axis. Find P.
ii. $Q(4, -2)$ and $Q'(4, 2)$ are a point and its image. In which axis was it reflected?
iii. $R(5, 3)$ is reflected in the X-axis, and its image is reflected in the X-axis again. Where does it end up? Explain why.
iv. Which points are their own image in the X-axis? Which point is its own image in both axes?

4 How far does it move? [4]

i. $A(3, 4)$ is reflected in the X-axis. Find A' and the length of AA'.
ii. Where does AA' cross the X-axis? How far is that point from A and from A'?
iii. $B(-2, 5)$ is reflected in the Y-axis. Find the length of BB'.
iv. A point in the 1st quadrant has x-coordinate 2. It is 6 units from its image in the X-axis. Find the point.

5 A slide and a parallelogram [4]

i. Draw $AB = 4$ cm across the page and construct $\triangle ABC$ above it, with $\angle A = 90^\circ$ and $AC = 3$ cm. Extend AB past B, and at B construct an arrow BK, 5 cm long, at 45° to the extended line, pointing up. Translate the triangle by the arrow BK.
ii. Measure $B'C'$, and check it with Pythagoras' theorem.
iii. Join AA', BB' and CC'. What do you notice? What kind of four-sided figure is $ABB'A'$? Give a reason.
iv. Describe the arrow that takes $\triangle A'B'C'$ back to $\triangle ABC$.

6 Reflection, translation or neither? [4]

Say whether each is a reflection, a translation or neither. Give a reason.

a. A rook moved three squares forward on a chessboard
b. The print of a left shoe beside the print of the right shoe of the same pair
c. A page of print held up to the light and read from the back
d. A wheel of a bicycle riding along a road

7 The clock in the mirror [4]

A clock with no numbers, only marks, is seen in a mirror hung beside it. The mirror flips it left to right, about the line through 12 and 6.

i. The clock shows 3 o'clock. What time does it seem to show in the mirror?
ii. It shows 2 o'clock. What time does the mirror seem to show?
iii. In the mirror the clock seems to show 4 o'clock. What time is it really?
iv. At which o'clock times does the mirror show the right time?

8 Think: the shortest way to the river [4]

A goat at $A(1, 4)$ walks straight to the river, which runs along the X-axis, drinks, and walks straight on to its shed at $B(10, 8)$.

i. Plot A, B and A', the image of A in the X-axis.
ii. Draw the straight line $A'B$. Where does it cross the X-axis? Call that point P.
iii. Explain why $AP = A'P$. Find AP and PB with Pythagoras' theorem, and the length of the whole walk $A \to P \to B$.
iv. If the goat drinks at $Q(1, 0)$ instead, is its walk longer or shorter? Explain why no point on the river gives a shorter walk than P.

9 Make your own [3]

i. Write a point and its image in the X-axis that are 10 units apart.
ii. Draw a triangle and an arrow, and draw the image of the triangle translated by the arrow.
iii. Write the corners of a four-sided figure that is its own image in the Y-axis.