Unit 16 · Solid Objects

HARDER PRACTICE QUESTIONS

Full marks 35

Write your answers on separate paper. Show your working. Take the circumference of a circle as 3.14 times its diameter. In questions 2 to 5 and 8, every solid has flat faces and no holes.

1 Spot the errors [4]

Aayusha wrote these four. Each one is wrong. Correct it, and say what went wrong.

i. A cuboid 4 cm by 3 cm by 2 cm is a regular solid, because all its corners are right angles.

ii. An octahedron has 8 vertices, because *okto* means eight.

iii. A drum has one circular face and one curved surface.

iv. An icosahedron has 20 triangles with 3 edges each, so it has 60 edges.

2 Working backwards [4]

Use $V - E + F = 2$.

i. A solid has 10 vertices and 15 edges. How many faces does it have?

ii. A solid has 9 faces and 16 edges. How many vertices does it have?

iii. Nishant counted a solid and wrote 7 vertices, 12 edges and 6 faces. Can his count be right? Give a reason.

iv. A solid has as many faces as vertices, and 10 edges. Find its number of faces.

3 Counting without counting [4]

A dodecahedron has 12 faces, each a regular pentagon, and 3 faces meet at every vertex.

i. Find its number of edges without counting them one by one.

ii. Find its number of vertices the same way.

iii. A ball is stitched from 12 pentagons and 20 hexagons. Every seam (edge) joins two patches, and 3 patches meet at every corner. Find its numbers of edges and corners.

iv. Check your numbers for the ball with $V - E + F = 2$. Is the ball's shape a regular solid? Give a reason.

4 Angles at a corner [4]

Each angle of an equilateral triangle is 60°, of a square 90°, of a regular pentagon 108° and of a regular hexagon 120°.

i. Mijash says 6 equilateral triangles can meet at a corner of a solid, because $6 \times 60°$ makes a whole turn. Is he right? Give a reason.

ii. How many regular pentagons can meet at a corner of a solid? Give a reason.

iii. At every corner of a solid, 2 squares and 1 equilateral triangle meet. Find the sum of their angles. Can they rise into a point?

iv. Which of these can be a corner of a solid: 3 regular hexagons; 1 square and 2 regular hexagons; 4 squares?

5 Cutting off corners [4]

Yangchen makes a tetrahedron from potato. With a knife, she slices off one corner with a flat cut close to the vertex.

i. What shape is the new face the cut makes? Give a reason.

ii. Find the numbers of vertices, edges and faces of the new solid.

iii. Check your numbers with $V - E + F = 2$. Is the new solid regular?

iv. She slices off all four corners of another tetrahedron in the same way. Find V, E and F, and check them.

6 Cones from one circle [4]

Bhumika cuts circles of radius 10 cm from paper to make cones. When she rolls one, its curved edge becomes the rim round the cone's open end.

i. How far is it round the whole circle?

ii. She cuts away a quarter and rolls the rest into a cone. How far is it round the rim of its open end, and how wide across is that end?

iii. From another circle she cuts away half. How wide across is this cone's open end? Is it taller or shorter than the first cone?

iv. She wants a cone 12 cm tall from one of these circles. Can she make one? Give a reason.

7 One card, two tubes [4]

Utsab has a rectangle of card 62.8 cm long and 31.4 cm wide. He can roll it into a tube either way, then close each end with a circle.

i. He rolls it so that the 62.8 cm side goes round. How wide across must each circle be? How tall is the tube?

ii. He rolls a second card the other way, so that the 31.4 cm side goes round. How wide across must its circles be? How tall is this tube?

iii. For the first tube, his friend cuts circles 18 cm across. What goes wrong?

iv. How many plane surfaces, curved surfaces, vertices and curved edges does each closed tube have?

8 Think: four triangles at every corner [4]

A solid is made of F equal equilateral triangles, with 4 of them meeting at every corner.

i. Explain why it has $3F \div 2$ edges and $3F \div 4$ vertices.

ii. Copy and complete this table.

F	**4**	**8**	**12**	**16**
E				
V				
$V - E + F$				

iii. Which of these numbers of triangles can close up a solid? Name the solid.

iv. Look at the last row. What is $V - E + F$ for any F? Explain why no other number of triangles works.

9 Make your own [3]

i. Draw a solid of your own with flat faces, not a cube, a cuboid or a regular solid. Count its vertices, edges and faces, and check that $V - E + F = 2$.

ii. Choose the length of a rectangle of card, and find how wide across the circles must be to close a tube rolled from it.

iii. From a circle of radius 10 cm, what part must be cut away to make a cone whose open end is 5 cm across?