Unit 10 · Indices

HARDER PRACTICE QUESTIONS · ANSWER KEY

Full marks 35

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Spot the errors

ANSWER U · 4

i. x^4. Dividing powers of the same base **subtracts** the indices: x^{6-2}. She divided them.

ii. **72.** The bases are different, so the law doesn't join them: $8 \times 9 = 72$. (6^5 is 7,776.)

iii. **5.** The 0 belongs to a alone: $5 \times a^0 = 5 \times 1 = 5$ (for $a \neq 0$).

iv. $\frac{1}{16}$. A minus in the index means 1 over the power: $\frac{1}{4^2} = \frac{1}{16}$. It never makes the number negative.

MARKING SCHEME

✓ i. x^4: subtract the indices.

✓ ii. 72: different bases can't be joined.

✓ iii. 5: only a has the index 0.

✓ iv. $\frac{1}{16}$: a negative index turns the power over.

2 Which is bigger?

ANSWER U · 4

a. $2^5 = 32$ and $5^2 = 25$: $\mathbf{2^5}$.

b. $2^{10} = 1024$ and $10^3 = 1000$: $\mathbf{2^{10}}$, just.

c. $3^{-2} = \frac{1}{9}$ and $2^{-3} = \frac{1}{8}$. An eighth is more than a ninth: $\mathbf{2^{-3}}$.

d. $(-2)^5 = -32$ and $(-3)^3 = -27$. On the number line -27 is to the right of -32: $\mathbf{(-3)^3}$.

MARKING SCHEME

✓ a. 2^5: 32 against 25.

✓ b. 2^{10}: 1,024 against 1,000.

✓ c. 2^{-3}: $\frac{1}{8}$ is more than $\frac{1}{9}$.

✓ d. $(-3)^3$: -27 is more than -32.

3 The missing index

ANSWER A · 4

a. $n + 3 = 9$, so $n = \mathbf{6}$.

b. $9 = 3^2$, so $7 - n = 2$ and $n = \mathbf{5}$.

c. Only the index 0 gives 1: $n = \mathbf{0}$.

d. $32 = 2^5$, so $\frac{1}{32} = \frac{1}{2^5} = 2^{-5}$: $n = \mathbf{-5}$.

MARKING SCHEME

✓ a. 6.

✓ b. 5.

✓ c. 0.

✓ d. −5.

4 One number, many powers

ANSWER

i. $64 = \mathbf{2^6 = 4^3 = 8^2}$.

ii. $4^3 = 64 = 2^6$, so $4^3 \times 2^4 = 2^6 \times 2^4 = 2^{6+4} = \mathbf{2^{10}}$ (1,024).

iii. $27 \times 27 = 3^3 \times 3^3 = 3^6$ and $9 \times 9 = 3^2 \times 3^2 = 3^4$, so $3^6 \div 3^4 = \mathbf{3^2}$ (9).

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iv. **16** ($2^4 = 4^2$), **64** ($2^6 = 4^3 = 8^2$) and **81** ($3^4 = 9^2$). **A · 4**

MARKING SCHEME

✓ i. 2^6, 4^3, 8^2.

✓ ii. 2^{10}.

✓ iii. 3^2.

✓ iv. 16, 64 and 81.

5 Folding paper

ANSWER **A · 4**

i. Each fold doubles: $2^5 = $ **32 sheets**.

ii. $2^{10} = 1024$ sheets: $1024 \times 0.1 = 102.4$ mm $\approx$ **10 cm**.

iii. $2^{20} = 2^{10} \times 2^{10} \approx 1000 \times 1000 = 10{,}00{,}000$ sheets: $10{,}00{,}000 \times 0.1 = 1{,}00{,}000$ mm = **about 100 m**. (Exactly, $2^{20} = 10{,}48{,}576$: about 105 m.)

iv. $2^{30} \approx 1000 \times 1000 \times 1000$ sheets: $10^9 \times 0.1 = 10^8$ mm $= 10^5$ m = **about 100 km**. (No one can fold real paper that often: each fold needs a sheet twice as long.)

MARKING SCHEME

✓ i. $2^5 = 32$ sheets.

✓ ii. About 10 cm.

✓ iii. About 100 m.

✓ iv. About 100 km.

6 Break it into primes

ANSWER **A · 4**

a. $6^4 = 2^4 \times 3^4$, so $\dfrac{2^4 \times 3^4 \times 2^{-1}}{3^3 \times 2^2} = 2^{4-1-2} \times 3^{4-3} = 2^1 \times 3^1 = \mathbf{6}$.

b. $12 \times 18 = 2^2 \times 3 \times 2 \times 3^2 = 2^3 \times 3^3$ and $27 = 3^3$. The numbers give $2^3 \times 3^{3-3} = 8$. The letters give $a^{5+2-6} = a^1$ and $b^{3+1-4} = b^0 = 1$. The answer is $\mathbf{8}a$.

MARKING SCHEME

✓ a. Prime factors, $6^4 = 2^4 \times 3^4$.

✓ a. 6.

✓ b. The numbers give 8.

✓ b. $8a$: $b^0 = 1$.

7 Put in the values

ANSWER **A · 4**

a. $2^{-1} = \mathbf{\dfrac{1}{2}}$.

b. $(-1)^3 + 3^{-1} = -1 + \dfrac{1}{3} = \mathbf{-\dfrac{2}{3}}$.

c. $(2-1)^3 = 1^3 = \mathbf{1}$.

d. $6^{-1} \times 3^2 = \dfrac{1}{6} \times 9 = \mathbf{\dfrac{3}{2}}$.

MARKING SCHEME

✓ a. $\dfrac{1}{2}$.

✓ b. $-\dfrac{2}{3}$.

✓ c. 1.

✓ d. $\dfrac{3}{2}$.

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8 Think: adding up the doubles

ANSWER HA · 4

i. **7, 15, 31.**

ii. Each sum is **one less than the next power of 2**: $1 = 2^1 - 1$, $3 = 2^2 - 1$, $7 = 2^3 - 1$, $15 = 2^4 - 1$, $31 = 2^5 - 1$.

iii. $2^{10} - 1 = 1024 - 1 = \mathbf{1{,}023}$.

iv. $2^5 = 2^4 \times 2$, so $2^5 - 2^4 = 2 \times 2^4 - 2^4 = 2^4$ (check: $32 - 16 = 16$). So if a sum is $2^4 - 1$, adding the next power, 2^4, gives $2^4 + 2^4 - 1 = 2^5 - 1$: again one less than the next power. Each power of 2 is the next one minus itself, so the pattern never stops.

MARKING SCHEME

- ✓ i. 7, 15, 31.
- ✓ ii. One less than the next power of 2.
- ✓ iii. 1,023.
- ✓ iv. $2^4 + 2^4 = 2^5$, so the pattern carries on.

9 Make your own

ANSWER A · 3

For example:

i. $2^4 \times 2^6$; $2^3 \times 2^3 \times 2^4$; $2^{12} \times 2^{-2}$.

ii. $5^7 \div 5^7 = 5^0 = 1$.

iii. 3^0, 1^5, $(-1)^2$.

Any correct answers are right.

MARKING SCHEME

- ✓ i. Three products adding to the index 10, one negative.
- ✓ ii. Equal indices, giving 5^0.
- ✓ iii. Three powers equal to 1.

See it at Science Playground

Exhibits and plants at the center, Bhaktapur, that go with this unit. Book a class visit at science-playground.com/booking.

- **Growing Dominoes** (exhibit): If each domino is 1.5 times the one before, two steps make 1.5^2, three more make 1.5^3, so five steps make $1.5^2 \times 1.5^3 = 1.5$I times the first.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	8	23	4	**35**

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