Unit 4 · Rational Number

HARDER PRACTICE QUESTIONS · ANSWER KEY

Full marks 34

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Spot the errors

ANSWER U · 4

i. They are the **same number**, -0.75. The additive inverse of $-\frac{3}{4}$ is $\frac{3}{4}$: $-\frac{3}{4}+\frac{3}{4}=0$.

ii. The reciprocal keeps the sign: $\mathbf{-\frac{5}{2}}$. $(-\frac{2}{5})\times(-\frac{5}{2})=1$, but $(-\frac{2}{5})\times\frac{5}{2}=-1$.

iii. A block of two digits repeats, so it goes over **99**: $0.\overline{36}=\frac{36}{99}=\frac{4}{11}\cdot\frac{36}{100}=0.36$ stops.

iv. $6=2\times3$: the 3 makes it **recurring**, $\frac{1}{6}=0.1666\ldots=0.1\overline{6}$. Only a denominator with no prime factors but 2 and 5 terminates.

MARKING SCHEME

- ✓ i. They are equal; the additive inverse is 3/4.
- ✓ ii. −5/2: the reciprocal keeps the sign.
- ✓ iii. 36/99 = 4/11.
- ✓ iv. Recurring: 6 has the factor 3.

2 Terminating or recurring?

ANSWER A · 4

a. Already in lowest terms; $40=2\times2\times2\times5$: **terminating** (0.175).

b. $\frac{9}{75}=\frac{3}{25}$, $25=5\times5$: **terminating** (0.12). The 3 in 75 cancels.

c. $\frac{21}{56}=\frac{3}{8}$, $8=2\times2\times2$: **terminating** (0.375). The 7 in 56 cancels.

d. Already in lowest terms; $48=2\times2\times2\times2\times3$: the 3 stays, so **recurring** ($0.2291\overline{6}$).

MARKING SCHEME

- ✓ a. Terminating: 40 has only 2s and 5s.
- ✓ b. 3/25, terminating.
- ✓ c. 3/8, terminating.
- ✓ d. Recurring: 48 has the factor 3.

3 In order

ANSWER A · 4

As decimals: -0.7; $-\frac{2}{3}=-0.666\ldots$; $-0.\overline{6}=-0.666\ldots$; $-\frac{5}{8}=-0.625$; $-\frac{3}{4}=-0.75$.

$\mathbf{-\frac{2}{3}}$ **and** $\mathbf{-0.\overline{6}}$ **are equal**: $0.\overline{6}=\frac{6}{9}=\frac{2}{3}$.

Of two negative numbers, the one further from 0 is smaller: $\mathbf{-\frac{3}{4}, -0.7, -\frac{2}{3}=-0.\overline{6}, -\frac{5}{8}}$.

MARKING SCHEME

- ✓ Each number as a decimal.
- ✓ $-\frac{2}{3}=-0.\overline{6}$.
- ✓ The furthest from 0 is the smallest.
- ✓ $-\frac{3}{4}, -0.7, -\frac{2}{3}=-0.\overline{6}, -\frac{5}{8}$.

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4 Recurring into fractions

ANSWER HA · 4

a. $3.\overline{18} = 3 + 0.\overline{18} = 3 + \frac{18}{99} = 3 + \frac{2}{11} = \mathbf{\frac{35}{11}}$.

b. Three digits repeat: $\frac{405}{999}$. Divide by 27: $\mathbf{\frac{15}{37}}$.

c. $10x = 1 + 0.\overline{6} = 1 + \frac{2}{3} = \frac{5}{3}$. So $x = \frac{5}{3} \div 10 = \frac{5}{30} = \mathbf{\frac{1}{6}}$. Check: $1 \div 6 = 0.1666\ldots$ ✓.

MARKING SCHEME

✓ a. 35/11.

✓ b. 15/37.

✓ c. $10x = 1\frac{2}{3} = \frac{5}{3}$.

✓ c. $x = \frac{1}{6}$.

5 Between two negatives

ANSWER A · 3

With 48ths: $-\frac{1}{3} = -\frac{16}{48}$ and $-\frac{1}{4} = -\frac{12}{48}$. Between them are $-\frac{15}{48}$, $-\frac{14}{48}$ and $-\frac{13}{48}$, that is $-\frac{5}{16}$, $-\frac{7}{24}$ and $-\frac{13}{48}$.

In order: $\mathbf{-\frac{1}{3}}, \mathbf{-\frac{5}{16}}, \mathbf{-\frac{7}{24}}, \mathbf{-\frac{13}{48}}, \mathbf{-\frac{1}{4}}$.

Any three correct numbers are right (the average $-\frac{7}{24}$ is one).

MARKING SCHEME

✓ Both written with a large enough common denominator.

✓ Three numbers between them.

✓ All five in ascending order.

6 Name the property

ANSWER U · 4

i. **Commutative property of multiplication**: $-\frac{3}{7}$ and $\frac{5}{2}$ swap places.

ii. **Associative property of multiplication**: the first two are grouped together.

iii. **Inverse property of multiplication**: $\frac{2}{5}$ and $\frac{5}{2}$ are reciprocals, so their product is 1.

iv. **Identity property of multiplication**: multiplying by 1 leaves a number as it was.

MARKING SCHEME

✓ i. Commutative.

✓ ii. Associative.

✓ iii. Inverse (multiplicative inverses).

✓ iv. Identity (1 is the multiplicative identity).

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7 A meter of ribbon

ANSWER A · 4

i. $\frac{1}{8} = \mathbf{0.125\ m}$; $\frac{1}{12} = 0.08333\ldots = \mathbf{0.08\overline{3}\ m}$.

ii. Only those whose prime factors are 2s and 5s: **2, 4, 5, 8 and 10**.

iii. **Yes**: 500 mm, 250 mm, 200 mm, 125 mm and 100 mm.

iv. For example **16**: $\frac{1}{16} = 0.0625$ m terminates, but it is 62.5 mm. (32 and 64 work too, at 31.25 mm and 15.625 mm; 20, 25, 40 and 50 don't, since they give whole millimeters.)

MARKING SCHEME

✓ i. 0.125 m and $0.08\overline{3}$ m.

✓ ii. 2, 4, 5, 8, 10.

✓ iii. Yes, with the lengths in mm.

✓ iv. A number such as 16: 62.5 mm.

8 Make your own

ANSWER A · 3

i. For example $\frac{3}{12} = \frac{1}{4} = 0.25$ (terminating: the 3 cancels; $\frac{6}{12}$ and $\frac{9}{12}$ work too), and $\frac{1}{12} = 0.08\overline{3}$ (recurring).

ii. For example -2: its reciprocal is $-\frac{1}{2}$, and $-\frac{1}{2} > -2$ (closer to 0). Any number less than -1 works.

MARKING SCHEME

✓ i. A terminating twelfth (top a multiple of 3).

✓ i. A recurring twelfth.

✓ ii. A negative number less than -1, checked.

9 Think: add the tops, add the bottoms

ANSWER HA · 4

i. **Yes**: $0.333\ldots < 0.4 < 0.5$.

ii. $\frac{5}{7} = 0.714\ldots$, and $0.666\ldots < 0.714\ldots < 0.75$: **yes**, it is between them.

iii. **No.** To add, the denominators must be made the same: $\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$. Her shortcut finds a number **between** the two, not their sum (the sum is bigger than both).

iv. **No.** Halfway is the average: $\frac{1}{2}\left(\frac{1}{3} + \frac{1}{2}\right) = \frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$, about 0.417, not 0.4.

MARKING SCHEME

✓ i. Yes, checked with decimals.

✓ ii. 5/7, between them.

✓ iii. No: the sum is 5/6.

✓ iv. No: halfway is 5/12.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	8	18	8	**34**

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