Unit 1 · Set

HARDER PRACTICE QUESTIONS

Full marks 34

Write your answers on separate paper. Give a reason for each answer. Write each element of a set once, and use $\subset$ for a proper subset and $\subseteq$ for an improper one, as the unit does.

1 Spot the errors [4]

Kushal wrote these four lines. Each has one mistake. Write each line correctly, and say what was wrong.

i. $\{0\}$ is the empty set, because 0 means nothing.
ii. $\{2\} \in \{1, 2, 3\}$
iii. $\{a, b\} \subset \{b, a\}$
iv. The set of the grains of sand on all the beaches of the world is infinite.

2 Empty, finite or infinite? [4]

Say whether each set is empty, finite or infinite. For a finite set, list it and give its number of elements.

a. $\{x : x \text{ is a natural number and } x + 5 = 3\}$
b. {two-digit numbers whose digits multiply to give 0}
c. {even numbers greater than one million}
d. {prime numbers that are even and greater than 2}

3 One rule, three universal sets [4]

The rule is "the factors of 36". Write the set it gives when the universal set is:

i. $U =$ {natural numbers less than 10};
ii. $U =$ {even numbers};
iii. $U =$ {odd numbers greater than 10}.
iv. Which two of your sets are equivalent but not equal?

4 Element or subset? [4]

$P = \{2, 3, 5, 7\}$. Copy each line and fill in $\in$ or $\notin$ when the left side is an element, $\subset$ or $\subseteq$ when it is a set. If none of them fits, write "none".

a. 5 ___ P
b. $\{5\}$ ___ P
c. 9 ___ P
d. $\{7, 5, 3, 2\}$ ___ P
e. $\{\}$ ___ P
f. $\{1, 2\}$ ___ P

5 A thali of your choice [4]

A restaurant serves dal-bhat with any choice of the extras $E =$ {achar, papad, curd, sweet}: all of them, some of them or none.

i. Each choice of extras is a subset of E. How many different choices are there? Explain how you know without listing them all.
ii. How many of the choices have curd in them?
iii. How many choices have exactly two extras? List them.
iv. Which choice is the improper subset of E? Which choice is the empty set?

6 Pairs that change with U [4]

$U = \{\text{natural numbers from 1 to } n\}$. In it, $A = \{\text{multiples of 4}\}$ and $B = \{\text{multiples of 6}\}$.

i. Take $n = 12$. List A and B. Are they equal, equivalent or neither?
ii. For which values of n from 1 to 12 are A and B equivalent?
iii. For which of those values are A and B also equal?

7 Equal, in disguise [3]

Which of these sets are equal to {2, 3, 5}? List each one to find out.

a. {prime factors of 60}
b. $\{x : x \text{ is a factor of 30, greater than 1 and less than 6}\}$
c. {odd prime numbers less than 7}

8 Make your own [3]

$U = \{\text{natural numbers up to 10}\}$. Write three sets A, B and C, all made from U, so that:

- $A \sim B$ but $A \neq B$;
- $C \subset A$ and $C \subset B$;
- C is not empty.

Show that every rule holds.

9 Think: always, sometimes or never? [4]

Is each statement always true, sometimes true or never true? Give an example or a reason for each.

i. Two equivalent sets are equal.
ii. An infinite set has a proper subset that is also infinite.
iii. The empty set is a proper subset of itself.
iv. If $A \subset B$ and $B \subset C$, then $A \subset C$.