Unit 1 · Set

HARDER PRACTICE QUESTIONS · ANSWER KEY

Full marks 34

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Spot the errors

ANSWER U · 4

i. **{0} is not empty**: it has one element, the number 0. The empty set is { } or ∅.
ii. **{2}** ⊂ **{1, 2, 3}** (or 2 ∈ {1, 2, 3}). {2} is a set, so it takes ⊂ ; only the element 2 takes ∈ .
iii. **{***a*, *b***}** ⊆ **{***b*, *a***}**. The two sets are equal (order doesn't matter), so {*a*, *b*} leaves nothing out: it is the **improper** subset.
iv. The set is **finite**. There are an enormous number of grains, but the counting would end: a set is infinite only if the counting could never end.

MARKING SCHEME

✓ i. {0} has one element, 0.
✓ ii. {2} ⊂ {1, 2, 3}: a set takes ⊂ .
✓ iii. ⊆ : the sets are equal, so the subset is improper.
✓ iv. Finite: very big, but the counting ends.

2 Empty, finite or infinite?

ANSWER A · 4

a. **Empty.** A natural number is at least 1, so $x + 5$ is at least 6 and can never be 3.
b. A product is 0 only when one of the digits is 0. The tens digit of a two-digit number can't be 0, so the ones digit is: {10, 20, 30, 40, 50, 60, 70, 80, 90}, **finite, 9 elements**.
c. **Infinite.** After any even number comes a bigger one: 1,000,002, 1,000,004, … never end.
d. **Empty.** Every even number greater than 2 has 2 as a factor, as well as 1 and itself, so it is not prime.

MARKING SCHEME

✓ a. Empty: $x + 5$ is at least 6.
✓ b. {10, 20, …, 90}, 9 elements.
✓ c. Infinite.
✓ d. Empty: every even number above 2 has the factor 2.

3 One rule, three universal sets

ANSWER A · 4

The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36. Only those in *U* count.

i. **{1, 2, 3, 4, 6, 9}**
ii. **{2, 4, 6, 12, 18, 36}**
iii. The odd factors are 1, 3 and 9, and none is greater than 10: **{ }**, the empty set.
iv. **The sets in i and ii**: 6 elements each, but 1 is in the first and not in the second (12 is in the second and not the first).

MARKING SCHEME

✓ i. {1, 2, 3, 4, 6, 9}.
✓ ii. {2, 4, 6, 12, 18, 36}.
✓ iii. The empty set.
✓ iv. i and ii: 6 elements each, different elements.

4 Element or subset?

ANSWER

a. **5** ∈ *P*: 5 is an element.
b. **{5}** ⊂ *P*: a set with one element, and *P* has more.

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c. **$9 \notin P$**: 9 is not prime. U · 4

d. **$\{7, 5, 3, 2\} \subseteq P$**: the same four elements, so it equals P.

e. **$\{\,\} \subset P$**: the empty set leaves every element out.

f. **None.** {1, 2} is a set, so only $\subset$ or $\subseteq$ could fit; but 1 is not in P, so {1, 2} is not a subset of P at all.

MARKING SCHEME

✓ a. $\in$ and c. $\notin$.

✓ b. $\subset$ and e. $\subset$.

✓ d. $\subseteq$.

✓ f. None: 1 is not in P.

5 A thali of your choice

ANSWER HA · 4

i. The subsets of {achar, papad, curd} are 8. Each of them can be taken with the sweet or without it, which doubles the count: **16 choices**.

ii. Every choice without curd (8 of them, the subsets of the other three extras) becomes one with curd when curd is added: **8 choices**.

iii. **6**: achar and papad, achar and curd, achar and sweet, papad and curd, papad and sweet, curd and sweet.

iv. The improper subset is **all four extras**, E itself. The empty set is **dal-bhat with no extras**.

MARKING SCHEME

✓ i. 16, by doubling 8 (or by listing).

✓ ii. 8 choices with curd.

✓ iii. The 6 pairs.

✓ iv. All four extras; no extras.

6 Pairs that change with U

ANSWER HA · 4

i. $A = \{4, 8, 12\}$ and $B = \{6, 12\}$: 3 elements and 2, so **neither** equal nor equivalent.

ii. Count the multiples up to each n.

n	1–3	4–5	6–7	8–11	12
elements of A	0	1	1	2	3
elements of B	0	0	1	1	2

They are equivalent for **$n = 1, 2, 3, 6$ and 7**.

iii. For $n = 1$, 2 and 3 both sets are **empty**, so they are equal: $A = B = \{\,\}$. For 6 and 7, $A = \{4\}$ and $B = \{6\}$: equivalent only.

MARKING SCHEME

✓ i. {4, 8, 12} and {6, 12}: neither.

✓ ii. The counts of each set worked out for each n.

✓ ii. $n = 1, 2, 3, 6, 7$.

✓ iii. $n = 1, 2, 3$: both empty.

7 Equal, in disguise

ANSWER A · 3

a. $60 = 2 \times 2 \times 3 \times 5$, so the prime factors are **{2, 3, 5}: equal**. (Each 2 is written once.)

b. The factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30. Greater than 1 and less than 6 leaves out 1 and 6: **{2, 3, 5}: equal**.

c. 2 is not odd: **{3, 5}: not equal**, and not even equivalent.

MARKING SCHEME

✓ a. {2, 3, 5}: equal.

✓ b. {2, 3, 5}: equal.

✓ c. {3, 5}: not equal.

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8 Make your own

ANSWER A · 3

For example $A = \{1, 2, 3\}$, $B = \{1, 2, 4\}$ and $C = \{1, 2\}$.

- A and B have 3 elements each, so $A \sim B$; 3 is in A and not in B, so $A \neq B$.
- 1 and 2 are in both A and B, and each leaves something out, so $C \subset A$ and $C \subset B$.
- C has 2 elements.

Any answer is right if every rule holds. C can only use elements that A and B share.

MARKING SCHEME

- ✓ A and B with the same number of elements, not the same elements.
- ✓ A non-empty C whose elements are all in both, leaving something out of each.
- ✓ Every rule checked.

9 Think: always, sometimes or never?

ANSWER HA · 4

i. **Sometimes.** $\{1, 2\} \sim \{2, 1\}$ and they are equal, but $\{1, 2\} \sim \{a, b\}$ and they are not.

ii. **Always.** Leave out one element of an infinite set: what remains still never runs out, and it leaves something out. For example, the even numbers are an infinite proper subset of the natural numbers.

iii. **Never.** A proper subset must leave at least one element out, and $\{\ \}$ has nothing to leave out. Its only subset is itself, the improper one.

iv. **Always.** Every element of A is in B, and every element of B is in C, so every element of A is in C. And C has an element that B hasn't, which A can't have either, so A leaves something of C out.

MARKING SCHEME

- ✓ i. Sometimes, with an example each way.
- ✓ ii. Always, with a reason or the even numbers.
- ✓ iii. Never: the empty set has nothing to leave out.
- ✓ iv. Always, with a reason.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	8	14	12	**34**

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