Unit 19 · Transformation

STUDY NOTES

Key terms

Transformation: moves a figure to a new position by a fixed rule.

Object: the figure we start with; **image**: the new figure; the image of A is A' ("A-dash").

Reflection: flips a figure over a line, the **axis of reflection** (line of reflection, mirror line).

Translation: moves every point the **same distance** (**magnitude**) in the **same direction**.

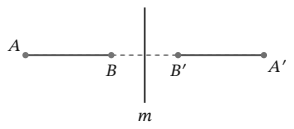
Rotation: a turn about a point (Grade 8).

Reflection

- fold, punch through both layers, open: the holes are the **same distance** from the fold, joined at **right angles** to it
- object and image: same distance from the axis, on **opposite sides**; **congruent**
- the image is flipped: clockwise $ABC \to$ anticlockwise $A'B'C'$
- a point **on** the mirror is its own image

To reflect by hand:

1. from each corner, a line to the mirror **at right angles**
2. carry it on the same distance past the mirror: the image point
3. join the images in the same order



AB reflected in the mirror line m.

- P 3 cm in front of $m \to P'$ 3 cm behind: $PP' = 6$ cm

Reflection on a graph

$(x, y) \to (x, -y)$	x stays, sign of y changes **Use:** reflection in the **X-axis**
$(x, y) \to (-x, y)$	y stays, sign of x changes **Use:** reflection in the **Y-axis**

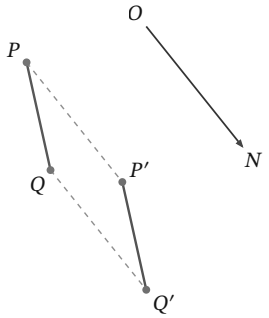
- X-axis: $A(4, 5) \to A'(4, -5)$; $P(-3, 2) \to P'(-3, -2)$: straight down across the axis
- Y-axis: $B(-3, 4) \to B'(3, 4)$; $M(-4, -5) \to M'(4, -5)$: straight across, same height
- image given, object asked: reflect back: $T'(3, -4) \to T(3, 4)$
- point to image = twice its distance from the axis: $TT' = 4 + 4 = 8$; $P(4, 5)$, $P'(-4, 5)$: $PP' = 8$

Translation

- slide an eraser along a line: AA', BB', CC', DD' are **equal and parallel**
- give **both** distance and direction: 3 cm up ≠ 3 cm right; an **arrow** gives both (its length, its way)
- the image is congruent, **not** turned or flipped: angles unchanged

To translate by an arrow:

1. from each point, a line **parallel to the arrow**, as long as it, pointing the same way
2. its end is the image; join the images



PQ translated by the arrow ON: PP' and QQ' parallel to ON and as long.

Reflection or translation?

	Reflection	Translation
image	flipped	same way up
points move	different distances (near the mirror, hardly)	all the same distance
lines to images	at right angles to the mirror, cut in half by it	equal and parallel to the arrow

Back to the ambulance

- AMBULANCE on the bonnet in mirror writing: a rear-view mirror flips it back, so the driver ahead reads it the right way round
- driving along a straight road: a **translation**
- a rubber stamp (मोहर): letters cut as mirror images; stamping along a line: translation

COMMON MISTAKES

✗ image of $(5, -2)$ in the Y-axis = $(5, 2)$
✓ that is the X-axis image; Y-axis: $(-5, -2)$.

✗ the image anywhere 3 cm on the other side
✓ on the line through the point at right angles to the mirror.

✗ a translation given as "move it 3 cm"
✓ give the direction too.

Science Playground, Katunje, Bhaktapur
https://science-playground.com/learn/grade-7-math/transformation