Unit 16 · Solid Objects

STUDY NOTES

Key terms

Face: a flat surface of a solid; **edge**: a line where two faces meet; **vertex** (plural **vertices**): a corner, where edges meet.

Regular solid: every face the **same regular polygon** (equal sides, equal angles), and the **same number of faces** at every vertex.

Tetrahedron: 4 equilateral triangles (*tetra*, four; *hedra*, face).

Octahedron: 8 equilateral triangles (*okto*, eight).

Skeleton model: only the edges (sticks) and vertices (joints); **hollow model**: only the thin shell.

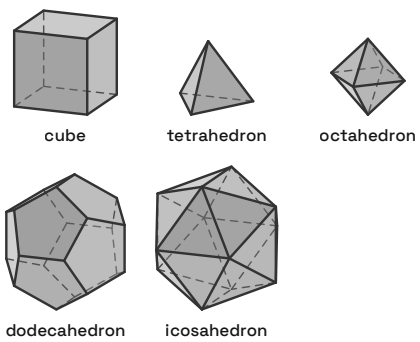
Cone: one vertex (its **apex**), one circular base, one curved surface.

Cylinder: two equal, **parallel** circular faces and one curved surface; no vertex.

Curved surface: one that bends; a ruler laid the wrong way touches it at one point.

The five regular solids

Solid	Faces	At each vertex	V	E	F
Tetrahedron	triangles	3	4	6	4
Cube	squares	3	8	12	6
Octahedron	triangles	4	6	12	8
Dodecahedron	pentagons	3	20	30	12
Icosahedron	triangles	5	12	30	20



The five regular solids: one regular polygon for every face, the same number at every vertex.

- exactly **five**; also called the **Platonic solids**; Euclid proved there is no sixth
- cube ↔ octahedron swap V and F (8, 6); dodecahedron ↔ icosahedron (20, 12); tetrahedron its own partner
- octahedron: like two square pyramids base to base; **no square face** inside, only a ring of 4 edges
- **not regular**: a cuboid that isn't a cube (rectangles); two tetrahedra glued face to face (6 equal triangles, but 3 faces at the top and bottom points, 4 round the middle)

Euler's formula

$V - E + F = 2$	V vertices, E edges, F faces **Use:** every solid with flat faces and no holes

- know two, find the third: tetrahedron $V = 4$, $E = 6$: $4 - 6 + F = 2 \rightarrow F = 4$
- octahedron, $V = 6$: first recall $E = 12 \rightarrow F = 8$
- same rule written as $F + V = E + 2$

Counting edges and vertices

- each edge belongs to **two** faces: edges = (faces × sides of a face) ÷ 2
- vertices = (faces × corners of a face) ÷ (faces at each vertex)
- icosahedron: $20 \times 3 = 60 \rightarrow$ **30** edges; $60 \div 5 =$ **12** vertices
- check: $12 - 30 + 20 = 2$ ✓

Skeleton models

- tetrahedron: **6** straws, **4** joints; octahedron: **12** straws, **6** joints (a square of 4, then 4 up, 4 down); cube: 12 sticks, 8 joints
- the faces are the gaps between the straws
- all triangles → **keeps its shape** (a triangle can't change shape unless a side changes length); a skeleton cube **leans** unless each face gets a diagonal stick
- so bridges, towers and roofs are braced with triangles

Why only five

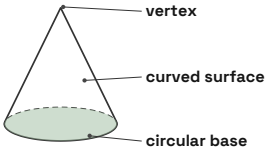
- the face angles round a vertex must add up to **less than 360°**; at least 3 faces at a vertex

Faces at a vertex	Sum	Solid
3 triangles	$3 \times 60° = 180°$	tetrahedron
4 triangles	$240°$	octahedron
5 triangles	$300°$	icosahedron
6 triangles	$360°$: **flat**, like floor tiles	none
3 squares	$270°$	cube
3 pentagons	$3 \times 108° = 324°$	dodecahedron
3 hexagons	$3 \times 120° = 360°$: flat	none

- 4 squares: $360°$, flat; 4 pentagons: $432°$, too many
- 10 triangles, 5 at a vertex: $E = 3F \div 2 = 15$, $V = 3F \div 5 = 6$; $6 - 15 + 10 = 1 \neq 2 \rightarrow$ can't close
- only $F = 20$ works with 5 at a vertex: $E = 30$, $V = 12$

Cone

- one vertex (apex), one circular base, one curved surface; the base's circle is one **curved edge**; no straight edge

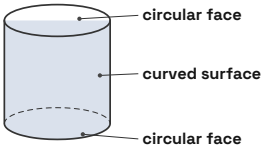


A cone's parts.

- to draw: slanted circles as ovals, the back half of the bottom one dashed; the sloping sides touch the base oval
- hollow cones: ice-cream cone, birthday cap, ढडिया (*dhadiya*, bamboo fish trap)
- funnel's top: curved surface only (no base, no vertex)
- **paper cone**: cut a circle, cut away a part, bend until edge OA lies on OB; the center O becomes the **vertex**
- more cut away → **narrower and taller**; the radius becomes the sloping side, so the height is less than the radius
- radius 10 cm: ¼ cut away → 15 cm across, 6.6 cm high; ¾ cut away → 5 cm across, 9.7 cm high
- nothing cut away: a flat circle can't bend into a cone without creasing

Cylinder

- two equal, parallel circular faces, one curved surface, **no vertex**; rolls in a straight line



A cylinder's parts.

- not a cylinder: a bucket (wider at the top)
- **paper cylinder**: roll card ABCD until AC lies on BD; paste a circle at each end
- each circle's **circumference = the card's length**; card 31.4 cm → diameter $31.4 \div 3.14 =$ **10 cm** (compass opened 5 cm)
- circle too big: sticks out; too small: a gap

	Cone	Cylinder
circular faces	1	2 (parallel)
curved surfaces	1	1
vertices	1	0

Where it is used

- **dice**: 4, 6, 8, 12 and 20 faces; every face alike, so each number equally likely
- **methane** (biogas, about half to four fifths of it): a tetrahedron, carbon in the middle, hydrogen at the 4 corners
- **crystals**: fluorite grows as cubes, sometimes octahedra; many **viruses** (herpes): an icosahedron shell
- cylinders: gas, water, oil kept and carried; cones: funnels, ice-cream cones

COMMON MISTAKES

✗ an octahedron has 8 vertices
✓ 8 **faces**; 6 vertices, 12 edges.

✗ icosahedron: $20 \times 3 = 60$ edges
✓ each edge counted twice: 30.

✗ equal triangle faces, so regular
✓ also the same number of faces at every vertex.

✗ a cuboid is regular (right angles)
✓ its faces aren't all the same square.

✗ a cylinder has one circular face
✓ two; the cone has one.

✗ 6 triangles at a corner make a point
✓ $360°$ lies flat.