Unit 14 · Plane Figures

STUDY NOTES

Key terms

Vertex: a corner of a triangle (plural vertices).

Rough sketch: a quick freehand triangle, named, with the given measurements on it; always first.

Parallelogram: four sides, **opposite sides parallel**.

Rectangle: a parallelogram with four right angles.
Square: a rectangle with four equal sides.

Hypotenuse h: the side opposite the right angle; always the **longest**.

Perpendicular p and **base** b: the two sides that make the right angle.

Triangles: names

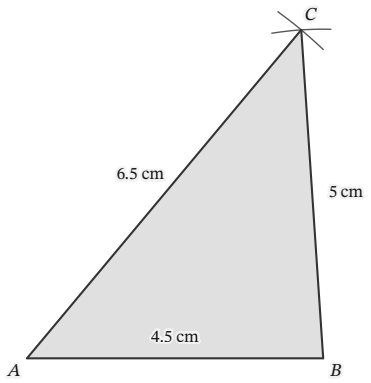
By sides	By angles
equilateral: all three equal	acute-angled: all three under 90°
isosceles: two equal	right-angled: one exactly 90°
scalene: none equal	obtuse-angled: one over 90°

- the three angles add to **180**°
- isosceles and right-angled: yes (90°, 45°, 45°); equilateral and obtuse: no (all 60°)

Constructing triangles

Three well-chosen parts fix a triangle; the lines and arcs cross at **one** point.

- **SAS** (two sides, the angle between): $PQ = 5.6$, $\angle Q = 60°$, $QR = 4.5$
  1. draw QP
  2. 60° at Q
  3. cut 4.5 cm on the arm at R
  4. join PR (≈ 5.1 cm)
- **ASA** (a side, the angles at its ends): $AB = 5.2$
  1. 75° at A, 60° at B on the same side
  2. arms meet at C
  3. $\angle C = 45°$
- **SSS** (three sides)
  1. draw $AB = 4.5$
  2. arc 6.5 cm from A
  3. arc 5 cm from B cuts it at C: the only point that far from both
- **AAS** (a side, an angle on it, the angle opposite): $XY = 5$, 45° at X, 75° at $Z \rightarrow$ angle at $Y = 180° - (45° + 75°) =$ **60**° first; then as ASA



SSS: $AB = 4.5$ cm, $BC = 5$ cm, $CA = 6.5$ cm, two arcs crossing at C.

- **don't fix one triangle**: AAA (shape, not size); SSA (can give two: the second side swings like a door)
- RHS (right angle, the side opposite it, one more side): fixes one
- **any two sides together are longer than the third**: 3, 4, 8 cm: arcs don't meet ($3 + 4 < 8$)

Parallelogram, rectangle, square

	Parallelogram	Rectangle	Square
opposite sides equal	✓	✓	✓
all sides equal			✓
opposite angles equal	✓	✓	✓
all angles 90°		✓	✓
diagonals bisect each other	✓	✓	✓
diagonals equal		✓	✓
diagonals cross at 90°			✓
diagonals bisect the corners (45°)			✓

- **verify by experiment**: two figures of each kind, measure sides, angles and the diagonals' parts

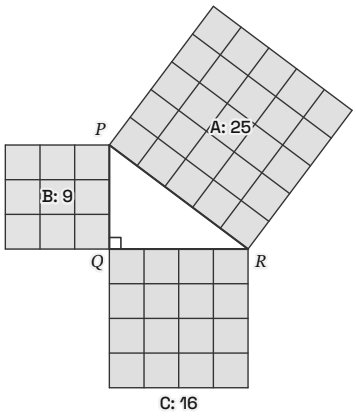
(AO, OC, BO, OD), fill a table, write the conclusion

- every square is a rectangle; every rectangle a parallelogram: what is true of all parallelograms is true of both
- parallelogram: two angles next to each other add to 180° (65°, 115°); its two diagonals are not equal (unless it is a rectangle)
- a leaning frame stays a parallelogram; its diagonals are equal only when upright (a rectangle)
- why a rectangle's diagonals are equal: $\triangle PQR$ and $\triangle QPS$ have the same two sides and right angle between (SAS), so the same third side
- book cover: rectangle
- carrom board: square
- cloth cut on a slant: parallelogram

Pythagoras' theorem

$h^2 = p^2 + b^2$	h hypotenuse, p perpendicular, b base **Use:** any right-angled triangle
$a^2 + b^2 = c^2$	c the hypotenuse, a, b the other two **Use:** the same rule, other letters

- the square on the hypotenuse = the squares on the other two together



Squares on the sides of a 3-4-5 triangle: 9 + 16 = 25.

- 3, 4 cm → squares of 9 and 16 small squares; on PR: **25** → PR = 5 cm
- four equal triangles in a square: leave $a^2 + b^2$ uncovered, slid round leave c^2
- **the other way round**: if the sides fit $h^2 = p^2 + b^2$, the angle opposite the longest side is 90°
- test: 13, 12, 5: $13^2 = 169 = 144 + 25$ → right-angled, the right angle opposite 13 cm
- missing side: $5^2 = PQ^2 + 4^2 \rightarrow PQ^2 = 9 \rightarrow PQ =$ **3 cm**
- 6 and 8 → $36 + 64 = 100 \rightarrow h = 10$ cm (3, 4, 5 doubled)
- 3, 4, 6: $9 + 16 = 25 < 36$ → the corner is wider than 90°
- known long before Pythagoras (Samos, about 2,500 years ago): Baudhayana Sulbasutra; Plimpton 322 (Babylon, about 1800 BC: 119, 120, 169)

Back to the mason

- rope of 12 equal spaces as 3, 4, 5: $3^2 + 4^2 = 5^2 \rightarrow$ right angle; three sides fix the triangle, so it can't take another shape
- bigger, still square: 6, 8, 10 or 9, 12, 15
- diagonals: four walls with opposite sides equal can still lean; **equal diagonals** → a rectangle, every corner square
- 4 m by 3 m: each diagonal 5 m only when upright

Where it is used

- ladders, ramps; a screen 16 by 12 is 20 across (256 + 144 = 400)
- 3 km east and 4 km north: 5 km apart

COMMON MISTAKES

✗ hypotenuse of 3 and 4 = 7
✓ square, add, then square root: $\sqrt{25} = 5$.

✗ 100 given as the hypotenuse of 6 and 8
✓ 100 is h^2; $h = 10$.

✗ the hypotenuse taken as any side
✓ the side opposite the right angle, the longest.

✗ AAS: 75° drawn at Y
✓ 75° is at Z: find the angle at Y first (60°).

✗ three angles fix a triangle
✓ only its shape; any size fits.

✗ a parallelogram's diagonals are equal
✓ only a rectangle's.

✗ sides 3, 4, 8 cm drawn as a triangle
✓ 3 + 4 < 8: no such triangle.