Unit 13 · Lines and Angles

STUDY NOTES

Key terms

Angle: two arms from one point, the **vertex**; measured in degrees; a full turn 360°.

Bisect: cut an angle in half.

Adjacent angles: same vertex, **one common arm**, on **opposite sides** of it (side by side, no overlap).

Vertically opposite angles: opposite each other where two lines cross ("vertical" = at the vertex); always **equal**.

Complementary angles: sum **90**°; each is the **complement** of the other.

Supplementary angles: sum **180**°; each is the **supplement**; they need not touch (70° and 110°).

Linear pair: two adjacent angles on a straight line; always supplementary.

Kinds of angle

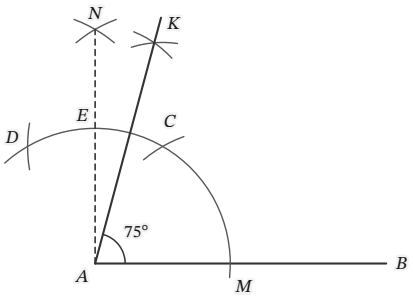
Kind	Size
acute	less than 90°
right	exactly 90°
obtuse	between 90° and 180°
straight	exactly 180°
reflex	between 180° and 360°

- reflex: measure the small angle, take it from 360° (70° → 290°)

Angles with a compass

- one step of the radius round an arc = **60**°; two steps 120°; three 180°
- bisect: needle on the two points where **the angle's own arms** cut the arc; two arcs, one radius; join the vertex to the crossing
- from Grade 6
  - 30° = half of 60°
  - 45° = half of 90°
  - 90° = halfway between 60° and 120°

New angle	Halfway between
15°	0° and 30° (60° halved twice)
75°	60° and 90°
105°	90° and 120°
135°	90° and 180°
150°	120° and 180°

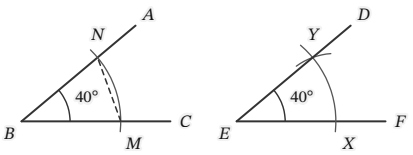


75°: arcs from E (the 90° arm) and C (the 60° arm) cross at K.

- 75°: from E and C; from E and D would give 105°
- halving can go on: 45° → 22½°
- angles at both ends of AB, on the same side, meet at C: $\angle ACB = 180° -$ the two (105° and 30° → **45**°); the two must add to less than 180°
- 20°, 40°, 10° **can't** be made with compass and ruler (Wantzel, 1837): use a protractor

Copying an angle

1. arc at Q, any radius: cuts the arms at M, N
2. **same radius** at A: cuts AB at X
3. open the compass to MN; from X, cut the arc at Y
4. join AY: $\angle CAB = \angle PQR$



Copying $\angle PQR$ at A: the same radius, then the gap MN.

- why: triangle AXY has the same three sides as QMN, so the same angles
- same radius matters: on a bigger arc the same angle has a bigger gap

Pairs of angles

- not adjacent: $\angle PQS$ contains $\angle PQR$ (they overlap) though they share QP
- **why vertically opposite angles are equal**: both are 180° − the same angle: $\angle AOP = 110° \rightarrow \angle POB = 70°$ and $\angle AOQ = 70°$
- proof credited to Thales of Miletus, about 2,600 years ago (Eudemus)
- complement = 90° − angle; supplement = 180° − angle: 33° → 57° and 147° (supplement is 90° more)
- supplement of 65° = 115°; complement 25°
- only angles under 90° have complements
- **c** before **s**, 90 before 180: **c**omplementary a **c**orner, **s**upplementary a **s**traight line

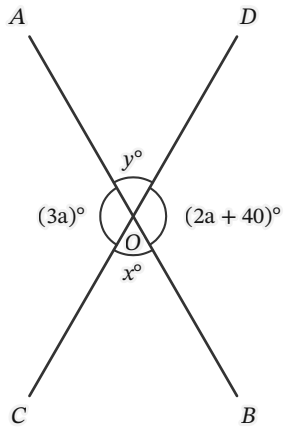
Three angle facts

1. vertically opposite angles are **equal**
2. adjacent angles on a straight line add to **180**°
3. angles round a point add to **360**° (180° above a line + 180° below)

Checking by experiment (each fact)

1. draw **two** figures of different sizes
2. measure the angles with a protractor
3. fill a table: the angles, their sum or which are equal
4. write the conclusion; a degree or so off is measuring, not a failure

Finding unknown angles



AB and CD cross at O.

- opposite angles equal: $3a = 2a + 40 \rightarrow a = 40 \rightarrow$ 120°; linear pair: $y = 60$; opposite: $x = y = 60$
- check: four angles at a crossing add to 360°
- on a line: $2a + 90 + a = 180 \rightarrow a = 30$
- round a point: $3x + 40 + x + 80 = 360 \rightarrow x = 60$
- linear pair: $7x + 3x = 180 \rightarrow x = 18$: 126° and 54°
- round a point: 100° and 140° → third 120°

Back to the scissors

- blades 40° → handles **40**° (vertically opposite)
- blades 70° → handles 70°: stay equal however the lines turn
- blade and the handle beside it: 180° − 40° = **140**° (linear pair)

Where it is used

- pliers: jaws open by the handles' angle
- frames: two 45° cuts make a 90° corner (complementary)
- crossing roads: measure one angle, know all four

COMMON MISTAKES

✗ complement of 65° = 115°
✓ that is the supplement; complement 25°.

✗ 75° by bisecting between E and D
✓ that gives 105°: use the 60° and 90° arms (C, E).

✗ copying with a different radius at A
✓ the same radius at both vertices.

✗ $\angle PQR$ and $\angle PQS$ (one inside the other) are adjacent
✓ adjacent angles sit side by side.

✗ 110° has a complement
✓ no: only angles under 90° have complements.

✗ opposite angles at a crossing add to 180°
✓ they are equal; the angles beside each other add to 180°.