Unit 12 · Equation, Inequality and Graph

STUDY NOTES

Key terms

Linear equation in two variables: two variables, each to the power 1, joined by "=": $x + y = 30$, $2x + y = 60$, $2x = 3y$.

Variable: a quantity that can change (a room's length x, breadth y); **constant**: fixed (300 m of wire).

Inequality: a sentence with $<$, $>$, $\leq$ or $\geq$; true for a whole stretch of numbers.

Trichotomy law: for any two numbers exactly one is true: $x = y$, $x < y$ or $x > y$.

Graph of a linear equation: the straight line through all its pairs of answers.

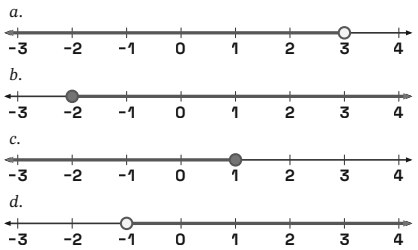
Linear equations in two variables

- "linear": from **line**
- not linear
  - $x^2 + y = 5$ (power 2)
  - $x + 4 = 8$ (one variable)
  - $2x + y > 6$ (inequality)
- one variable: one answer; two variables: **many pairs**
- know one number → put it in its letter's place → solve for the other
- potatoes x + onions $y = 30$ kg; $x = 20 \rightarrow y =$ **10 kg**
- fence 300 m round a rectangle: $2(x + y) = 300 \rightarrow x + y = 150$
- room: $x + y = 22$, length 15 → breadth **7 m**
- ages: $2x + y = 60$, Aayusha 10 → father **40**
- **a second fact**: put it in → one variable
  - boys double the girls: $x + y = 75$, $x = 2y \rightarrow 3y = 75 \rightarrow$ 25 girls, **50 boys**
  - mother three times her son: $3y + y = 40 \rightarrow$ **10** and **30**
  - two consecutive odd numbers: x and $x + 2$: $2x + 2 = 56 \rightarrow$ **27, 29**
- path 5 m round a pond x by y: outside $(x + 10)$ by $(y + 10)$: $2(x + y + 20) = 210 \rightarrow x + y = 85$
- **write what each letter stands for first**: "5 black pens" goes in place of y if y is black pens: $x + 5 = 17 \rightarrow$ 12 red

Inequality signs

Sign	Read	On the number line
$>$	greater than	**hollow** circle, arrow right
$<$	less than	**hollow** circle, arrow left
$\geq$	greater than or equal to	**filled** circle, arrow right
$\leq$	less than or equal to	**filled** circle, arrow left

- hollow: the end number is **not** included ($4 > 4$ false); filled: it is ($2 \geq 2$ true)



Reading a number line: a. $x < 3$; b. $x \geq -2$; c. $x \leq 1$; d. $x > -1$.

- the arrow points where the answers go: right for greater, left for less
- integers
  - $x > 4 \rightarrow \{5, 6, 7, \ldots\}$
  - $x \leq 1 \rightarrow \{1, 0, -1, \ldots\}$
  - $x \geq 2 \rightarrow \{2, 3, 4, \ldots\}$
- $-10 > -15$: -10 is to the right
- signs from Thomas Harriot's algebra book (printed 1631); the open end faces the **bigger** number

Rules of inequality

If $x > y$:

Do to both sides	Result	Axiom
add z	$x + z > y + z$	addition
subtract z	$x - z > y - z$	subtraction
× positive z	$xz > yz$	multiplication
÷ positive z	$x/z > y/z$	division
× negative z	$xz < yz$	multiplication, sign **turns**
÷ negative z	$x/z < y/z$	division, sign **turns**

- **× or ÷ by a negative turns the sign**: $> \leftrightarrow <$, $\geq \leftrightarrow \leq$
- why: $5 > 2$, $\times (-1) \rightarrow -5$ and -2; -5 is to the left: $-5 < -2$
- $6 > 3$, $\div (-3) \rightarrow -2 < -1$

Solving inequalities

1. do the same to both sides, as for an equation

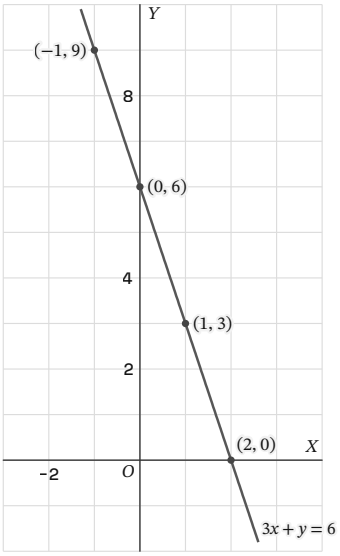
2. dividing by a negative: write the turned sign **in the same line**
3. check with one number from the answer (and one outside)

- $x + 2 > 5 \to x > 3$; integers {4, 5, 6, ...}
- $3x - 2 \leq -11 \to 3x \leq -9 \to x \leq -3$ ($\div$ 3, sign stays)
- $3 - 2x \leq 9 \to -2x \leq 6 \to x \geq -3$; check $x = 1$: $1 \leq 9$ ✓; $x = -4$: 11, not ≤ 9 ✓
- $5 - x > 2 \to -x > -3 \to x < 3$
- $-3x > 12 \to x < -4$, not $x > -4$ (test 0: $0 > 12$ false)
- $x - 4 < 8 \to x < 12$; whole numbers {0, 1, ..., 11}
- x on both sides: letters to one side, numbers to the other: $5x - 7 \leq 2x + 5 \to 3x \leq 12 \to x \leq 4$
- words: "13 subtracted from double a number is at most 3" $\to 2x - 13 \leq 3 \to x \leq 8$

Graph of a linear equation

1. make it $y = \ldots$: $3x + y = 6 \to y = 6 - 3x$
2. choose values of x, work out y
3. plot **at least three** points (a wrong one sticks out), draw the line

x	0	1	-1	2
$y = 6 - 3x$	6	3	9	0

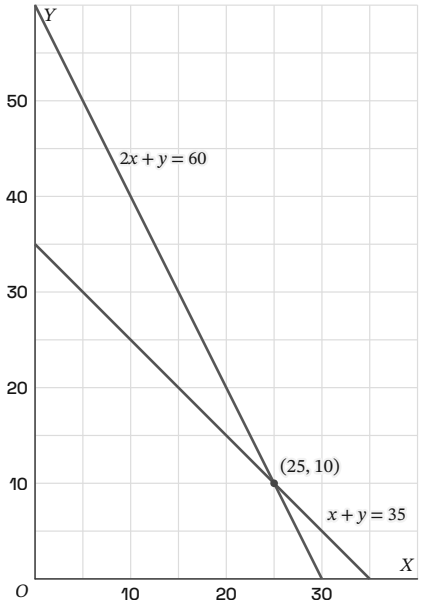


The table's points on one line: $3x + y = 6$.

- every point on the line is an answer, every answer is on the line
- $x + y = 11$ (balls): (0, 11), (2, 9), (4, 7), (6, 5) on one line; only whole-number points make sense for balls
- choose x to give whole numbers: $y = (3x - 1)/2$: odd x; $y = (6 - 4x)/3$: multiples of 3
- on $x + y = 4$? add the pair: (5, −1) yes; (2, 3) no

Back to the shop

- 2 pens + 1 copy = Rs 60: $2x + y = 60$: many answers (20, 20), (25, 10), (15, 30)...
- second sale: $x + y = 35$; one more pen, Rs 25 more $\to$ pen **Rs 25**, copy **Rs 10**
- on a graph: two lines **cross** at (25, 10); one equation: many answers, two together: usually one (Grade 8)



The two sales' lines cross at (25, 10): pen Rs 25, copy Rs 10.

Where it is used

- $F = 9/5 \times C + 32$: a straight-line graph; 100 °C = 212 °F
- budget line: $25x + 10y = 60$; what you can afford: $25x + 10y \leq 60$

COMMON MISTAKES

✗ 5 black pens put in place of x (red)
✓ each number in its own letter's place.

✗ $-3x > 12 \to x > -4$
✓ $\div$ a negative turns the sign: $x < -4$.

✗ $3x \leq -9 \to x \geq -3$
✓ $\div$ a positive keeps it: $x \leq -3$.

✗ $x > 4$ drawn with a filled circle at 4
✓ hollow: 4 is not included.

✗ only two points plotted
✓ plot three: a slip shows as a point off the line.

✗ $-15 > -10$
✓ -10 is to the right: $-10 > -15$.

Science Playground, Katunje, Bhaktapur
https://science-playground.com/learn/grade-7-math/equation-inequality-and-graph