Unit 11 · Algebraic Expression

STUDY NOTES

Key terms

Constant: a fixed value: 9, −4, ½.

Variable: a quantity that can change, written as a letter: x, y, a.

Term: constants and variables joined by × or ÷: 5, x, $7xy$; terms are separated by + and −.

Coefficient: the number multiplying the variables: 7 in $7xy$, −2 in $-2y$, 1 in x.

Like terms: the same variables with the same powers: $3x^2$, $-5x^2$; $-3ab$, $7ba$.

Binomial: two terms: $2x - 3$. **Trinomial**: three terms: $x^2 + 7x + 12$.

Remainder: what is left at the end of a division: 0, or a lower power than the divisor.

Formulas

$(a+b)^2 = a^2 + 2ab + b^2$	square of a sum **Use:** a square made longer
$(a-b)^2 = a^2 - 2ab + b^2$	square of a difference **Use:** b^2 is + in both
$(a+b)^2 - (a-b)^2 = 4ab$	the two linked **Use:** sum, difference, product
$a^2 + b^2 = (a+b)^2 - 2ab$	sum of squares **Use:** sum and product known

- $(a-b)^2 = (a+b)^2 - 4ab$; $(a+b)^2 = (a-b)^2 + 4ab$
- $(x+y+z)^2 = (x+y)^2 + 2(x+y)z + z^2$ = the three squares + twice each pair: $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
- **Dividend = Divisor × Quotient + Remainder** (as $23 = 4 \times 5 + 3$)

Grade 6 rules

- add or subtract **like terms only**: $3x^2 - 5x^2 = -2x^2$; $3x^2 + 3x$ stays
- multiply terms: multiply coefficients, **add** powers: $2x \times 3x^2 = 6x^3$
- divide terms: divide coefficients, **subtract** powers: $12x^5 \div 4x^2 = 3x^3$

Multiplying

- **each term of one × each term of the other**, then collect like terms
- binomial × binomial: $2 \times 2 = \mathbf{4}$ products; rectangle $(a+b)$ by $(x+y)$ = four parts: $ax + ay + bx + by$
- trinomial × binomial: $3 \times 2 = \mathbf{6}$ products
- in practice: break up one bracket: $(2x - y)(x + 2y - 3z) = 2x(\ldots) - y(\ldots)$
- the sign goes with its term: $-y \times (-3z) = +3yz$
- $(5x - 2y)(7x - 2y) = 35x^2 - 10xy - 14xy + 4y^2 = 35x^2 - 24xy + 4y^2$
- $(x+3)(x-5) = x^2 - 5x + 3x - 15 = x^2 - 2x - 15$
- leave a term with no like term ($-6xz$); order: highest power of the first letter first
- area of a plot = length × breadth: $(x + 2y)(3x - y) = 3x^2 + 5xy - 2y^2$ m^2
- **check**: put a number in before and after: $(5x-3)(3x+4) = 15x^2 + 11x - 12$; $x = 2$: $7 \times 10 = 70 = 60 + 22 - 12$ ✓

Dividing by a binomial

Like long division of numbers, terms in place of digits:

1. both in order, highest power first; like terms in one column
2. first term of what is left ÷ first term of the divisor → next term of the quotient
3. multiply the **whole divisor** by it, write it under the like terms
4. subtract: **change every sign** of that product and add
5. repeat until 0 or a lower power than the divisor (the remainder)

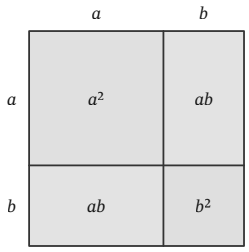
$$
\begin{array}{r|l}
x^2 + 7x + 12 & x + 4 \\
\hline
x^2 + 4x & x + 3 \\
\hline
3x + 12 & \\
3x + 12 & \\
\hline
0 &
\end{array}
$$

$(x^2 + 7x + 12) \div (x + 4) = x + 3$: a playground's breadth from its area and length.

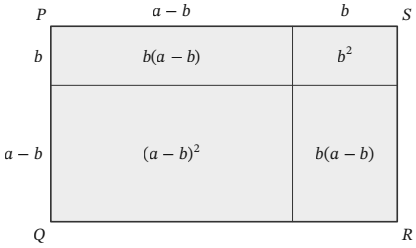
- check by multiplying: $(x+4)(x+3) = x^2 + 7x + 12$ ✓

- $(4x^2 - y^2) \div (2x - y) = 2x + y$
- $(2x^2 - 3x + 7) \div (2x - 1) = x - 1$, remainder **6**
- $(x^2 + 7x + 25) \div (x + 3) = x + 4$, remainder 13; $x = 40$: $43 \times 44 + 13 = 1905$ ✓
- **a missing side** = area ÷ the known side: $(35x^2 - xy - 12y^2) \div (7x + 4y) = 5x - 3y$; one factor from a product the same way
- missing terms: leave a gap: $(x^3 - y^3) \div (x - y) = x^2 + xy + y^2$
- $(16x^2 + 24xy + 9y^2) \div (4x + 3y) = 4x + 3y$: a square, $(4x + 3y)^2$

Squares from a cut square



$(a + b)^2$ from the cut square.



$(a - b)^2$: the square of side a with the strips off.

- side $a + b$ cut into four: a^2, **two** rectangles ab, b^2 (Euclid, *Elements* II.4, about 300 BC)
- $(a - b)^2$: from a^2 take two strips ab; the corner b^2 went twice → put it back once
- in words: square of the first + **twice the product** + square of the second
- $(x + 2)^2 = x^2 + 4x + 4$: the same
  - multiply out
  - the formula
  - a figure
- in your head
  - $12^2 = 100 + 40 + 4 = 144$
  - $88^2 = (90 - 2)^2 = 8100 - 360 + 4 = 7744$
  - $51^2 = 2601$
  - $99^2 = 9801$
- the first term squared whole: $(2y - 3)^2 = 4y^2 - 12y + 9$
- $(2x - 1/(3x))^2$: middle term $2 \times 2x \times 1/(3x) = 4/3$ → $4x^2 - 4/3 + 1/(9x^2)$
- **whole square form**: two terms squares, the third twice their product
  - $4x^2 - 20xy + 25y^2$: $(2x)^2$, $(5y)^2$, middle $2(2x)(5y)$ → $(2x - 5y)^2$
  - $x^2 - 2 + 1/x^2 = (x - 1/x)^2$
- $(2a + b)^2 - (2a - b)^2 = 8ab$: keep the second square in brackets

A sum and a product

- $x + 1/x = 6$: square → $x^2 + 2 + 1/x^2 = 36$ → $x^2 + 1/x^2 = 34$; $(x - 1/x)^2 = 36 - 4 = 32$
- $a + b = 8$, $ab = 12$: $a^2 + b^2 = 64 - 24 = 40$; $(a - b)^2 = 64 - 48 = 16$ → $a - b = \pm 4$
- back to the sum: $a^2 + b^2 = 17$, $ab = 4$ → $(a + b)^2 = 17 + 8 = 25$ → $a + b = \pm 5$
- a square has **two** square roots: give both unless told which is bigger

Back to the vegetable bed

- 10 m → 12 m a side: $(10 + 2)^2 = 100 + 40 + 4 = 144$ → **44 m²** new, not 4
- the brother counted only the corner b^2; the two strips (20 m² each) are most of it
- a side a made longer by b adds $2ab + b^2$

COMMON MISTAKES

✗ $(x + 3)(x - 5) = x^2 - 15$
✓ four products: $x^2 - 2x - 15$.

✗ $-y \times (-3z) = -3yz$
✓ the sign stays with its term: $+3yz$.

✗ subtracting $4x^2 - 2xy$ gives $-4x^2 - 2xy$
✓ change every sign: $-4x^2 + 2xy$.

✗ $(a + b)^2 = a^2 + b^2$
✓ $(3 + 4)^2 = 49$, $3^2 + 4^2 = 25$: add $2ab$.

✗ $(x - 3)^2 = x^2 - 6x - 9$
✓ $(-3)^2 = +9$: $x^2 - 6x + 9$.

✗ $(2y - 3)^2 = 2y^2 - 12y + 9$
✓ $(2y)^2 = 4y^2$.

✗ $(a - b)^2 = 16$, so $a - b = 4$
✓ $a - b = \pm 4$.