Unit 10 · Indices

STUDY NOTES

Key terms

Power: $a^n = a \times a \times \cdots$, n factors of a; read "a to the power n".

Base: the number multiplied: 3 in 3^5.

Index (**power**, **exponent**): how many times the base is a factor: 5 in 3^5; plural **indices**.

Index form: a product written as a power: $5 \times 5 \times 5 \times 5 \times 5 = 5^5$.

Laws of indices

$a^m \times a^n = a^{m+n}$	same base **Use:** multiplying: **add** the indices
$a^m \div a^n = a^{m-n}$	same base, $a \neq 0$ **Use:** dividing: top index − bottom index
$a^0 = 1$	$a \neq 0$ **Use:** any number to the index 0
$a^{-n} = \frac{1}{a^n}$	$a \neq 0$ **Use:** negative index: 1 over the power

- unsure? **write the factors out and count**: the law is what you see
- $2^3 \times 2^4$: 3 generations, then 4 more = 7 → $2^7 = 128$
- $5^5 \div 5^2$: two 5s below cancel two above → $5^3 = 125$
- lakh 10^5, crore 10^7: $10^7 \div 10^5 = 10^2$ → a crore = **100 lakh**; arab 10^9 = 100 crore

Why $a^0 = 1$

- $a^m \div a^m$: a number divided by itself is 1, and the law gives $a^{m-m} = a^0 \rightarrow a^0 = 1$
- going down the powers of 2 halves each step: 8, 4, 2, **1** (2^0), then 1/2 (2^{-1}), 1/4 (2^{-2})
- $7^0 = 100^0 = (3a)^0 = 1$; but $3a^0 = 3 \times 1 = 3$ (the 0 is on a only)

Negative index

- $2^1 \div 2^3 = 2/(2 \times 2 \times 2) = 1/2^2$, and the law gives $2^{-2} \rightarrow 2^{-2} = 1/4$
- the minus means **1 over**, never a negative number: $3^{-2} = 1/9$

Using the laws

- index form: the **whole** repeated factor is the base, in brackets: $(-3y)^6$, not $-3y^6$
- $(3xy)^5 = 3^5x^5y^5$: each factor of the base gets the index
- several bases: divide each by its own power: $3^5x^5y^5 \div 3^2x^2y^2 = 27x^3y^3$
- numbers: break into primes first: $10 \times 15/75 \rightarrow 2 \times 3 \times 5^2$ over 3×5^2
- a bracket can be the base: $(a+b)^3 \times (a+b)^5 = (a+b)^8$
- indices that cancel: $x^{a-b} \times x^{a+b} \times \cdots \times x^{c-a}$: add in pairs → $x^{2(a+b+c)}$; if $a+b+c=0$, $x^0 = 1$
- prime factors in index form
  - $128 = 2^7$
  - $243 = 3^5$
  - $625 = 5^4$
  - $343 = 7^3$
  - $1728 = 2^6 \times 3^3$
- signs: $(-2)^4 = +16$ (even), $(-5)^3 = -125$ (odd)
- a fraction as the base, in brackets: 1/3 seven times $= \left(\frac{1}{3}\right)^7$
- **substituting**: work out the index first, then the power: $x=1, y=2, z=-1 \rightarrow z^{xy} = (-1)^2 = 1$; $(xyz)^{-1} = (-2)^{-1} = -\frac{1}{2}$; $3^x \times 2^y = 12$

Back to the family tree

- 25 generations back: $2^{25} = 2^{10} \times 2^{10} \times 2^5 = 1024 \times 1024 \times 32 =$ **3,35,54,432** places
- more than Nepal's 2,91,64,578 people (2021); the same ancestor fills many places (**pedigree collapse**)

Where it is used

- 1 km = $10^3 \times 10^3$ mm = 10^6 mm
- earthquakes: one step of magnitude = waves about 10 times as tall; 7.8 vs 5.8 → about $10^2 = 100$ times

COMMON MISTAKES

✗ $2^5 = 2 \times 5 = 10$
✓ the index counts factors: $2 \times 2 \times 2 \times 2 \times 2 = 32$.

✗ $a^3 = 3a$
✓ $a \times a \times a$; $3a$ is $a + a + a$.

✗ $2^3 \times 2^4 = 2^{12}$; $y^9 \div y^3 = y^3$
✓ add: 2^7; subtract: y^6.

✗ $2^3 \times 3^4 = 6^7$
✓ different bases don't join: $8 \times 81 = 648$.

✗ $25^0 = 0$
✓ $25^0 = 1$.

✗ $3^{-2} = -9$
✓ 1 over 3^2: 1/9.