Unit 9 · Perimeter, Area and Volume

STUDY NOTES

Key terms

Perimeter of a triangle: the total length of its three sides, all the way round.

Total surface area: the area of all the faces of a solid: the paper that covers it exactly, no overlap; cm^2, m^2.

Net: a box cut along some edges and opened out flat: six rectangles in three equal pairs.

Cube: a cuboid with length, breadth and height equal.

Volume: the space a solid takes up, in unit cubes: cm^3, m^3.

Circumference: the length round a circle (its perimeter); a wheel rolls it in one turn.

Pi, π: circumference ÷ diameter, the same for every circle, a little more than 3.14; decimals never end or repeat (3.14159…).

Formulas

$P = a + b + c$	sides of a triangle **Use:** any triangle
$P = 3a$	three equal sides a **Use:** equilateral triangle
$P = 2a + b$	two equal sides a, third b **Use:** isosceles triangle
$A = 2(lb + bh + lh)$	length, breadth, height **Use:** surface area of a cuboid
$A = 6a^2$	edge a **Use:** surface area of a cube
$V = l \times b \times h$	cubic units **Use:** volume of a cuboid
$V = a^3$	edge a **Use:** volume of a cube
$c = \pi\, d = 2\pi\, r$	$d = 2r$; $\pi = \frac{22}{7}$ or 3.14 **Use:** circumference
$d = \frac{c}{\pi}$, $r = c \times \frac{7}{44}$	the second with $\pi = \frac{22}{7}$ **Use:** diameter, radius from c

Perimeter of a triangle

- sides named by small letters, each after the corner **opposite** it: a = BC, b = CA, c = AB
- 8 + 12 + 6 = **26 cm**; plot 15, 17, 12 m → wall **44 m**
- missing side = perimeter − the other two: 400 − (120 + 180) = **100 m**
- rounds of wire: perimeter × rounds: (10 + 12 + 14) × 5 = 180 m; at Rs 20 a meter **Rs 3,600**
- sticks inside a triangle add to the total, not to the perimeter: 22.5 + 7 + 7 = 36.5 cm
- **same unit first**: 1 m, 80 cm, 90 cm → 100 + 80 + 90 = **270 cm**
- **any two sides together are longer than the third**: 3, 4, 9 no triangle (3 + 4 < 9); 3, 4, 6 yes

Surface area of a cuboid

Faces	Each face	Pair
top and bottom	l by b	$2lb$
front and back	l by h	$2lh$
two ends	b by h	$2bh$

- 4 × 3 × 2 cm: 2(12 + 6 + 8) = **52 cm^2**; geometry box 15 × 7 × 3: 2(105 + 21 + 45) = **342 cm^2**
- **no lid**: five faces: $A = lb + 2bh + 2lh$
- missing breadth: 7,812 = 2(42b + 28b + 42 × 28) = 140b + 2,352 → b = 5,460/140 = **39 cm**

Surface area of a cube

- $l = b = h = a \rightarrow 2(a^2 + a^2 + a^2) = 6a^2$; or 6 square faces of a^2
- edge 3.5 cm: 6 × 12.25 = **73.5 cm^2**
- from the area to the edge: ÷ 6, then **square root**: $6a^2 = 54 \rightarrow a^2 = 9 \rightarrow a$ = **3 m**
- the same 64 cm^3 → **nearer a cube, less card**
  - 4 × 4 × 4 needs 96 cm^2 of card
  - 8 × 4 × 2: 112
  - 16 × 2 × 2: 136
  - 64 × 1 × 1: 258

Volume

- paint pot $15 \times 14 \times 13 =$ **2,730 cm³**; rice bin, edge 60 cm: **216,000 cm³** = 216 liters
- sugar store $3 \times 2 \times 1 =$ **6 m³**
- tank 600 liters, 200 cm long, 50 cm high → in m: $0.6 = 2 \times b \times 0.5 \rightarrow b =$ **0.6 m** = 60 cm
- length 3 × breadth, height 8, volume 864: $3x \times x \times 8 = 24x^2 = 864 \rightarrow x = 6 \rightarrow 18 \times 6 \times 8$; surface area $2(108 + 48 + 144) =$ **600 cm²**
- cube tank, edge 1.5 m: 3.375 m³ = **3,375 liters**
- 216,000 liters = 216 m³ → $a = \sqrt[3]{216} = 6$ m → surface area $6 \times 36 =$ **216 m²**
- from the volume to the edge of a cube: **cube root**
- **melted and remade**: the shape changes, the **volume stays**: gold $4 \times 2 \times 1 = 8$ cm³ → cube of edge 2 cm; silver cube, surface 150 → edge 5 → 125 cm³ = $l \times 5 \times 4 \rightarrow l =$ **6.25 cm**
- **how many jugfuls**: big volume ÷ small volume: pot $32 \times 16 \times 8 = 4{,}096$ ÷ jug $8^3 = 512 \rightarrow$ **8**
- **ground covered** by a solid standing on it: the area of one face: cube of edge 11 → **121 cm²**
- **cost** = area × rate: room $8 \times 6 \times 2 \rightarrow 152$ m² at Rs 50 = **Rs 7,600**; seats by **floor** area: 48 ÷ 0.5 = **96** students

Liters and cubic meters

- **1 liter** = a cube 10 cm each way = **1,000 cm³**
- **1 m³** = $100 \times 100 \times 100 = 1000000$ cm³ = **1,000 liters**; 1 liter = 1/1000 m³
- a 1,000-liter roof tank holds exactly 1 m³ (50 jars of 20 liters)
- m³ → liters: × 1,000; liters → m³: ÷ 1,000

Circumference

- thread round a coin about 2 cm across: about 6.28 cm → 6.28 ÷ 2 = **3.14**; every circle, any size
- Earth: 40,075 km round the equator ÷ 12,756 km across ≈ 3.14
- $d = 14$ cm → $r = 7 \rightarrow c = 2 \times 22/7 \times 7 =$ **44 cm**
- ring, $c = 7$ cm → $d = 7 \times 7/22 = 49/22 \approx$ **2.23 cm**
- pond $d = 56$ m: $c = 22/7 \times 56 = 176$ m; 5 rounds **880 m**
- cart wheel turns 100 times in 440 m → $c = 4.4$ m = 440 cm → $2 \times 22/7 \times r = 440 \rightarrow r =$ **70 cm**
- distance = number of turns × circumference; turns = distance ÷ circumference; a goat on a 14 m rope walks a circle of radius 14 m

Back to Science Day

- prize box $30 \times 20 \times 10$: $2(600 + 200 + 300) =$ **2,200 cm²** of paper, a little more for folds
- tank of 1,000 liters = 1 m³ → $a^3 = 1 \rightarrow$ edge **1 m**
- wheel clicking every meter: $c = 100$ cm → $d = 100/3.14 \approx$ **31.8 cm**

COMMON MISTAKES

✗ 1 m, 80 cm, 90 cm: 1 + 80 + 90 = 171
✓ same unit first: 270 cm.

✗ surface area in cm³
✓ an area: cm² or m².

✗ a box with no lid: $2(lb + bh + lh)$
✓ five faces: $lb + 2bh + 2lh$.

✗ cube with surface area 54: $a = 54/6 = 9$
✓ $a^2 = 9$, so $a = 3$.

✗ 216 cm² and 216 cm³ are the same thing
✓ faces and cubes: edge 2 cm gives 24 cm² but 8 cm³.

✗ 1 m³ = 100 liters
✓ 1,000 liters ($100 \times 100 \times 100$ cm³).

✗ a radius put into $c = \pi d$
✓ half the answer: use $2\pi r$ with a radius, πd with a diameter.

✗ people a room holds = volume ÷ space each
✓ space for people is floor area: 48 ÷ 0.5 = 96, not 96 ÷ 0.5 = 192.

✗ sides 3, 4 and 9 cm make a triangle
✓ 3 + 4 is not more than 9: no triangle.