Unit 8 · Unitary Method

STUDY NOTES

Key terms

Unitary method: find the value of **one** ("unit" means one), then of any number from it.

Direct variation: one quantity goes up (or down) **in the same proportion** as the other goes up (or down); their **ratio** stays the same.

Indirect (inverse) variation: as one goes up, the other goes **down** in the same proportion; their **product** stays the same.

Person-days: people × days, the total work: 1 person for 10 days = 2 people for 5 days.

Direct or indirect?

	Direct	Indirect
one goes up	the other goes **up**	the other goes **down**
stays the same	the ratio (value of one)	the product (total work)
to get to one	**divide**	**multiply**
then	**multiply**	**divide**
proportion	same order: $3/5 = 270/x$	turned upside down: $12/8 = x/20$
examples	books and price, kg and cost	workers and days, speed and time

- ask first: when one goes **up**, does the other go up or down?
- check the answer moves that way
- **neither**: both grow, but not in proportion: a girl 1.2 m at 8 won't be 2.4 m at 16; the unitary method can't answer

Direct variation

- 1 book Rs 50: 2 books 100, 3 books 150, … 6 books **300**; total ÷ books = 50 always
- 30 bananas Rs 225 → 1 banana 225/30 = Rs 7.50 → 12 bananas **Rs 90**
- 3 books Rs 270 → 1 book Rs 90 → 5 books **Rs 450**
- by proportion: $3/5 = 270/x \rightarrow 3x = 1350 \rightarrow x = 450$ (or $5/3 = x/270$)
- 4 kg tomatoes Rs 240 → 1 kg Rs 60 → 7 kg **Rs 420**

Indirect variation

- 1 worker 10 days, 2 workers 5, 5 workers 2: $1 \times 10 = 2 \times 5 = 5 \times 2 =$ **10 person-days**
- 12 men dig a field in 20 days → 1 man $20 \times 12 =$ 240 days → 8 men 240/8 = **30 days**
- by proportion: $12/8 = x/20 \rightarrow 8x = 240 \rightarrow x = 30$ (or $8/12 = 20/x$)
- check: $12 \times 20 = 240 = 8 \times 30$ ✓; fewer men take longer ✓
- food for 50 people for 54 days → 1 person 2,700 days → 60 people **45 days**
- bus 40 km/h for 3 hours = 120 km → at 60 km/h: 120/60 = **2 hours** (faster, less time)

Other kinds

- **two things, one price known**: 3 kg lentils + 2 kg sugar = Rs 540, sugar Rs 90/kg → lentils 540 − 180 = 360 → **Rs 120**/kg
- **change the units first**: 15 km an hour = 15,000 m in 60 min → **250 m** a minute
- **a table**: find the value of 1 kg first (10 kg Rs 1,250 → Rs 125), then fill each row from it
- mixed exercise: rows and columns of a square each +10% → chairs $\times 1.1 \times 1.1 =$ **+21%**, not 10% or 20%

Back to the ropai

- 12 people plant the field in 5 days → 1 person 60 days → in 3 days: 60/3 = **20 people**
- wages: $12 \times 5 = 60$ person-days last year, $20 \times 3 =$ 60 this year: **the same**
- the method assumes everyone works at the same speed

Where it is used

- price per kg or per liter = the value of one
- exchange rates: the value of one dollar, or of 100 Indian rupees
- kilometers per liter: the distance for one liter of fuel

COMMON MISTAKES

✗ 8 men take $20 \times 8/12 \approx 13$ days
✓ indirect: fewer men take longer: 240/8 = 30 days.

✗ workers and days: divide to get to one
✓ indirect: multiply to get to one (12×20), then divide.

✗ $3/5 = x/270$ for books and price
✓ keep the same order: $3/5 = 270/x$.

✗ height doubles when age doubles
✓ neither: not every pair that grows together is direct.

✗ 20 planters cost more wages
✓ the same 60 person-days of work, the same wages.