Unit 6 · Ratio and Proportion

STUDY NOTES

Key terms

Ratio $a : b$ (or a/b, $b \neq 0$): compares two quantities of the **same kind** by dividing; says **how many times** one is the other; read "a is to b".

Terms: a and b; the first is the **antecedent**, the second the **consequent**.

Simplest form (**lowest terms**): no number but 1 divides both terms.

Proportion: a statement that two ratios are equal: $1 : 3 = 4 : 12$.

In proportion: a, b, c, d with $a : b = c : d$, written $a : b :: c : d$, read "a is to b as c is to d".

Extremes: the outer terms a and d. **Means**: the inner terms b and c.

Rules

$a : b :: c : d \Rightarrow a \times d = b \times c$	product of the extremes = product of the means **Use:** test a proportion; find a missing term
$a : b = (a \times n) : (b \times n)$	the same $n \neq 0$ on both terms **Use:** equal ratios; simplest form (÷ the HCF)

Comparing two quantities

- **subtracting**: how much more: 60 − 30 = 30 kg
- **dividing** (a ratio): how many times: 30 kg : 60 kg = 1 : 2, half as much; the same at any size
- exercise book Rs 70, pencil Rs 10 → 70 : 10 = **7 : 1**; eraser Rs 5 : pencil = **1 : 2**
- **order matters**: pencil : book = 1 : 7, another ratio; ask "of what to what?"

Writing a ratio

1. put both quantities in the **same unit**
2. divide one by the other: the units cancel → **no unit**
3. divide both terms by the HCF → simplest form

- pencil: diameter 6 mm, length 18 cm = 180 mm → 6 : 180 = **1 : 30** (not 18 : 6 = 3 : 1)
- 2 kg : 500 g = 2000 g : 500 g = **4 : 1**, not 2 : 500
- kilograms with rupees: not a ratio at all (different kinds)
- buckets 9 liters and 15 liters → HCF 3 → **3 : 5**
- Rs 45 : Rs 120 = **3 : 8** (HCF 15)
- 1 hour : 40 min = 60 : 40 = **3 : 2**
- 1.2 m : 80 cm = 120 : 80 = **3 : 2**
- heights 145 cm : 165 cm = (5 × 29) : (5 × 33) = **29 : 33** (29 is prime)
- 100 Indian rupees = 160 Nepali rupees → numbers of rupees for the same value 100 : 160 = **5 : 8**

Ratio and fraction

	Fraction	Ratio
compares	a **part** with the **whole**	a **part** with the **other part**
circle, 3 of 5 parts shaded	3/5 shaded	shaded : not shaded = 3 : 2

- 3 : 2 does **not** mean 3/2 of the circle is shaded (more than the whole)
- fraction of the whole: add the terms: 3 + 2 = 5 → 3/5 and 2/5

Sharing in a ratio

1. add the terms: the number of equal parts
2. amount ÷ parts = one part
3. each share = its term × one part; check the shares add up

- Rs 6,000 in 5 : 7 → 12 parts → 1 part Rs 500 → **Rs 2,500** and **Rs 3,500**
- girls 180 of 450 → boys 270 → girls : all = 2 : 5, boys : all = 3 : 5, girls : boys = **2 : 3**

Comparing results

- write each as marks obtained : full marks, lowest terms, then a common denominator
- Utsab: Maths 75/100 = 3/4 = 9/12; Science 50/75 = 2/3 = 8/12 → **better in Maths**
- Sushant 25/30 = 5/6, Nirjala 20/24 = 5/6 → **equally good**: 25 : 30 :: 20 : 24

Proportion

- 25 : 30 :: 20 : 24: extremes 25 and 24, means 30 and 20; $25 \times 24 = 600 = 30 \times 20$
- why: a/b = c/d over the same denominator $b \times d$ $\rightarrow a \times d = b \times c$
- works both ways: equal products $\rightarrow$ in proportion
- 3, 4, 9, 12: $3 \times 12 = 36 = 4 \times 9 \rightarrow$ **in proportion**; or $3/4 = 9/12$
- sheets of 4 and 16 squares, 1 and 4 shaded: 1 : 3 = 4 : 12

Finding a missing term

1. product of the extremes = product of the means
2. solve for x
3. check: the two ratios simplify to the same

- $3 : 5 :: 12 : x \rightarrow 3x = 60 \rightarrow x =$ **20** (12 : 20 = 3 : 5)
- $4 : 7 :: x : 21 \rightarrow 7x = 84 \rightarrow$ **12**; $9 : x :: 3 : 8 \rightarrow 3x = 72 \rightarrow$ **24**
- sel roti, rice flour : sugar = $4 : 1 :: 5 : x \rightarrow 4x = 5 \rightarrow$ **1.25 kg** sugar
- word problems: like with like, in the same order
  - 6 kg : 8 kg :: Rs 540 : $x \rightarrow$ **Rs 720**
  - honey : water = 200 : 3,000 :: x : 600 $\rightarrow$ **40 ml** (the number of people doesn't matter; liters into ml first)
- both ratios in the **same order**: 2 : 3 :: 4 : 6, but 6 : 4 = 3 : 2

Back to the mortar

- 1 : 4 has no unit: one pan to four pans is the same mix as one bucket to four, measured with the **same** pan
- 3 pans of cement: $1 : 4 :: 3 : x \rightarrow x =$ **12 pans** of sand
- $2 : 8 = 1 : 4 (\div 2)$; $1 \times 8 = 4 \times 2 \rightarrow$ **the same mix**, a bigger batch
- one more pan of each: 2 : 5, not 1 : 4: to keep a ratio **multiply** both terms, never add the same number; a double batch is 2 : 8
- concrete M15: cement : sand : gravel = 1 : 2 : 4; M20: 1 : 1.5 : 3

COMMON MISTAKES

✗ 18 cm : 6 mm = 3 : 1
✓ same unit first: 180 mm : 6 mm = 30 : 1, so diameter : length = 1 : 30.

✗ 2 kg : 500 g = 2 : 500
✓ 2,000 g : 500 g = 4 : 1.

✗ buckets: 3 liters : 5 liters as the answer
✓ a ratio has no unit: 3 : 5.

✗ shaded : not shaded = 3 : 2, so 3/2 is shaded
✓ $3/(3 + 2) = 3/5$ of the whole.

✗ better in Maths because $75 > 50$
✓ full marks differ: compare $75/100 = 3/4$ with $50/75 = 2/3$.

✗ cement : sand = 1 : 4 :: 12 : 3
✓ keep the order: 1 : 4 :: 3 : 12.

✗ 1 : 4 $\rightarrow$ add 1 to each, 2 : 5, for a bigger batch
✓ multiply: 2 : 8.

✗ extremes 30 and 20 in 25 : 30 :: 20 : 24
✓ extremes are the outer terms, 25 and 24.